Proc. of Int. Conf. on Current Trends in Eng., Science and Technology, ICCTEST

Homotopy Analysis of MHD Slip-Flow with Heat and Mass Transer Due to a Point Sink

Chandrakant Guled[1] and Brijbhan Singh[3]
[1-2] Dr. Babasaheb Ambedkar Technological University, Lonere, Raigad, India
Email: chandrakant.guled@gmail.com, bbsingh@dbatu.ac.in

***Abstract*—These instructions In the present study, an analytical solution of the magnetohydrodynamic steady incompressible laminar boundary layer flow with a velocity slip on the wall-boundary of a cone due to a point sink at the vertex has been studied in the presence of heat transfer, mass transfer and magnetic parameters by using the homotopy analysis method (HAM). The HAM produces an analytical solution of the governing self-similar non-linear two-point boundary layer equations with great precision. Further, the effects of the suction/injection, velocity slip and magnetic parameters over the given flow-field have been discussed. The effects of Prandtl number on temperature and Schmidt number on concentration profiles have also been studied graphically. The present results have been compared numerically and graphically with the corresponding results obtained by other methods, an excellent agreement has been found between them. The accuracy in the result shows that the HAM aided by computing software like MATHEMATICA, Maple, etc. is very proficient and easily applicable technique for solving similarity equations with strong non-linearity. The analytical solution obtained by HAM is very near to the exact solution for a properly selected initial guess, auxiliary and convergence control parameters and for higher orders of deformations.**

***Index Terms*— MHD flow, homotopy analysis method (HAM), boundary layer flow, suction and injection, velocity slip.**

I. INTRODUCTION

The boundary layer flow in a cone due to a point sink at the vertex simulates the flow problems in nozzles wherein the hole at the vertex serves as the three-dimensional point sink. Despite the significance of such type of problems, the literature available pertaining to this kind of flow is not enough. However, a few relative studies in this connection have been carried out by many researchers. As the investigation of a boundary layer flow of an electrically conducting fluid on a cone due to a point sink with an applied magnetic field is significant in the study of conical nozzle or diffuser flow problems, many a research workers have given their valuable contributions to have an elaborate analysis of such type of problems. In this context, the contribution made by Rosenhead [1] is worthwhile mentioning. Rosenhead [1] presented the similarity solution for the heat transfer analysis of the axi-symmetric flow inside a cone due to a point sink. Ackerberg [2] presented the series solution for the converging motion of viscous fluid inside a cone. Choi and Wilhelm [3] made a thorough study of the self-similar magneto-hydrodynamic diffuser flows with induced magnetic fields. Takhar et al. [4] extended the problem for an electrically conducting fluid and discussed the effects of heat and mass transfer in the presence of an applied magnetic field. Eswara et al. [5] investigated the same problem for the transient case. Eswara and Bommaiah [6] re-investigated the problem by taking into account the temperature dependent fluid viscosity.

Grenze ID: 02.ICCTEST.2017.1.157

The objective behind the present work is to extend the problem of Takhar et al. [4] by taking into consideration the momentum slip boundary condition on the wall boundary of the cone. While studying the Falkner-Skan flow over a wedge with slip boundary conditions, Martin and Boyd [7] proved that the slip conditions on the wall boundary must be added to correctly predict the local wall shear stress and heat transfer rate in boundary layer analysis. This is particularly necessary for the rarefied flow conditions which have several physical applications such as in aerosol science [8], in microchannels [9] and in micro-nano-air vehicles [10]. The flow-slip and temperature jump effects over a specific wedge-shaped surface has been studied by Turkyilmazoglu [11]. Recently, Turkyilmazoglu [12] extended the flow model of Magyari [13] set up for moving convergent channel by taking into account the momentum and thermal slip boundary conditions on the boundary of the convergent channel. Inspired by the work of Turkyilmazoglu [12], in the present analysis, we have revisited the work of Takhar et al. [4] by including the slip conditions on the wall boundary of the cone. The problem, which is governed by non-linear equations with two-point boundary conditions, has been solved by using HAM. Symbolic computation software and high performance computers have been used to derive the analytic solutions. The flow characteristics have been analyzed numerically and graphically in order to study the effects of heat and mass transfer, slip flow and magnetic parameters on velocity, temperature and concentration profiles. Our results obtained by using HAM for $\lambda = 0$ have been found in excellent agreement with the corresponding results of Takhar et al. [4] who tackled the problem by using shooting technique in conjunction with the Runge-Kutta fourth order method. When λ is set to zero, the flow solutions in Ref. [4] are recovered.

The strength of HAM developed by Liao [14] in the year 1992 lies in the fact that it is proficient enough to lead to convergent analytic series solutions of strongly non-linear problems faster than any other existing methods, and is completely independent of the presence of small/large physical parameter/s in the problem [15]. This strength of HAM makes it superior to all other existing conventional perturbation methods such as Adomian decomposition method [16,17,18], δ-expansion method [19] and Lyapunov artificial small parameter method [20] which may not be valid for solving strong non-linear problems due to the divergent nature of their obtained solution series. Liao [21] also showed that these methods are simply special cases of the HAM. Likewise, He's homotopy perturbation method (HPM) [22,23] is also a special case of the HAM (cf. Liao [24] and Turkyilmazoglu [25]). On account of the above-mentioned facts, the HAM has been applied to an extensive variety of non-linear problems in branches of science and engineering ever since it was first introduced in the year 1992. The problems (see the References [26,27,28]) which mainly deal with the flows of non-Newtonian fluids and the problems (see References [29,30]) which mainly govern the non-linear problems with heat transfer and radiation effects, have been tackled by using the HAM).

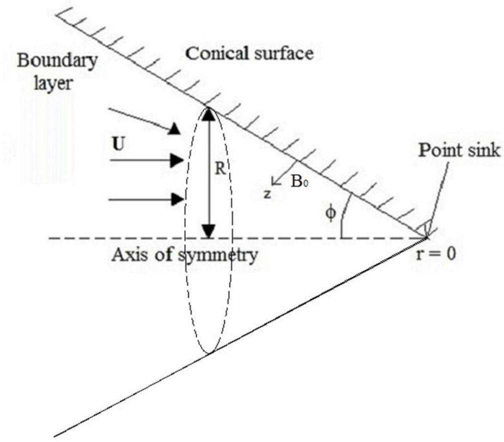

Figure 1:Schematicdiagram oftheflowmodel

II. Governing Equations

Here the steady, laminar and axi-symmetric boundary layer flow of an incompressible and electrically conducting fluid inside a cone at rest with a three-dimensional sink at its vertex has been considered by taking into consideration the slip-flow condition on the wall boundary of a cone. The magnetic field B_0 applied in z-direction has been taken as fixed relative to the fluid. The magnetic Reynolds number is assumed to be small so as to neglect the induced magnetic field as compared to the applied magnetic field. The temperature and concentration at the wall of the cone and at the free-stream have been maintained at constant level. To provide significance to the investigation, the effects of suction/injection, slip-flow and magnetic parameters have been included in the analysis. The terms representing the Hall and dissipation effects have been neglected. It is further assumed that the injected gas has a static temperature equal to the wall temperature, and it has the same physical properties as the boundary layer gas possesses.

The flow model has been presented in the schematic diagram given in the Fig. 1. The basic boundary layer equations governing the flow-field are therefore (cf. Takhar et al. [4]):

$$(ru)_r + (rw)_z = 0 \tag{1}$$

$$uu_r + wu_z = -\rho^{-1}p_r + \nu u_{zz} - \rho^{-1}\sigma B_0^2 u \tag{2}$$

$$uT_r + wT_z = \alpha T_{zz} \tag{3}$$

$$uC_r + wC_z = DC_{zz} \tag{4}$$

where

$$-\rho^{-1}p_r = UU_r + \rho^{-1}\sigma B_0^2 U, \quad U = -\frac{m_1}{r^2}\ , \qquad m_1 > 0 \tag{5}$$

Here the subscripts r and z denote derivatives with respect to r and z, respectively, and p represents the static pressure. The boundary conditions are given by

$$\begin{aligned} &u(r,0) = l_1(r)u_z(r,0), \quad w(r,0) = w_w, \quad T(r,0) = T_w, \quad C(r,0) = C_w \\ &u(r,\infty) = U, \quad T(r,\infty) = T_\infty, \quad C(r,\infty) = C_\infty\ . \end{aligned} \tag{6}$$

We now use the following similarity transformations in Eqs. (1)-(6):

$$\begin{aligned} &\eta = m_1^{\frac{1}{2}}z/(2\upsilon r^2)^{\frac{1}{2}}, \quad ru = \psi_z, \quad rw = -\psi_r, \quad \psi = -(2m_1\upsilon r)^{\frac{1}{2}}f \\ &u = Uf'(\eta), \quad w = \left({}^{m_1\upsilon}/_{2r^3}\right)^{\frac{1}{2}}(f - 3\eta f') \\ &(T - T_\infty)/\,(T_w - T_\infty\,) = g(\eta), \quad (C - C_\infty)/\,(C_w - C_\infty\,) = G(\eta) \end{aligned} \tag{7}$$

$$M = 2\sigma B_0^2 r^3\ /\ m_1\varrho\ , Pr = \ \upsilon/\alpha, Sc = \upsilon/D, K_w = w_w(2r^3/m_1\upsilon)^{1/2}, \lambda = l_1(r)/(2\upsilon r^3)^{\frac{1}{2}}\ .$$

Her r is the distance along the cone from the vertex and u is the corresponding velocity component along r direction, whereas z is the distance perpendicular to the cone; w is velocity component along z direction; R is radius of the cone given by $R = r\sin\phi$ where ϕ is the semi-vertical angle of the cone (see Fig. 1); ψ is the dimensional stream function and f represents the corresponding dimensionless stream function; C is the dimensional concentration and G is the corresponding dimensionless concentration; T is the dimensional temperature while g represents dimensionless temperature; σ, ϱ and υ are the electrical conductivity, density and kinematic viscosity, respectively; η represents similarity variable; B_0stands for magnetic field; D and α represent the binary diffusion coefficient and thermal diffusivity, respectively;m_1 is the strength of point sink; M is the magnetic parameter; U represents the inviscid flow velocity; K_w denotes the mass transfer parameter; $l_1(r)$ is slip factor;λ is slip parameter, and prime denotes derivatives with respect to η.

$$f''' - ff'' + 4(1 - f')^2 + M(1 - f') = 0 \tag{8}$$

$$g'' - Prfg' = 0 \tag{9}$$

$$G'' - ScfG' = 0 \tag{10}$$

Also, the boundary conditions (6) get reduced to

$$f(0) = K_w, \qquad f'(0) = \lambda f''(0), \quad f'(\infty) = 1 \tag{11}$$

$$g(0) = 1, \qquad g(\infty) = 0 \tag{12}$$

$$G(0) = 1, \qquad G(\infty) = 0 \tag{13}$$

where Sc and Pr are Schmidt and Prandtl numbers, respectively; the subscripts w denote conditions at the wall and ∞ denote conditions in the free stream; and prime denotes derivative with respect to η.

It is here to be noted that the above mentioned boundary layer approximation is not valid in the immediate neighbourhood of the hole (cf. Rosenhead [1], p.428). Also, for the mathematical analysis, the mass transfer parameter K_w has been treated as constant (cf. Takhar et al. [4]). Further, the magnetic parameter M can be treated as constant locally for fixed r, as it was first considered by Takhar et al. [31] and then by Takhar and Nath [32]. Moreover, the similarity reduction reported in this work is constant only when the slip parameter λ does not depend on r and hence requiring that $l_1(r)$ is proportional to $r^{3/2}$. The value of $f''(0)$ is of physical interest here, because it is associated with the local skin friction. Also, in a sink flow, $K_w < 0$ is refereed to as suction and $K_w > 0$ as injection (cf. Takhar et al. [4] and Schlichting and Gersten [33], pp. 294-298).

III. HOMOTOPYANALYSIS

Solution for skin friction (Eqs. (8) and (11))

In order to find the analytic solution of the Eq. (8) along with the boundary conditions given by Eq. (11), we first select the linear operator $\mathcal{L}$ as

$$\mathcal{L} = \frac{\partial^3}{\partial\eta^3} + \gamma\frac{\partial^2}{\partial\eta^2}\ , \tag{14}$$

and we choose q as an embedding parameter. We, now, construct the following zeroth-order deformation equation:

$$(1-q)\mathcal{L}[f(\eta,\hbar,\gamma,q) - f_0(\eta)] = q\hbar\aleph[f(\eta,\hbar,\gamma,q)]\ , \qquad \eta\in[0,+\infty), \hbar\neq 0, \gamma>0, q\in[0,1] \tag{15}$$

with boundary conditions

$$f(0,\hbar,\gamma,q) = K_w\ , f'(0,\hbar,\gamma,q) = \lambda f''(0), \qquad f'(\infty,\hbar,\gamma,q) = 1, \qquad \hbar\neq 0, \gamma>0, q\in[0,1] \tag{16}$$

where the prime denotes the partial derivative w.r.t. η and

$$\aleph[f(\eta,\hbar,\gamma,q)] = \frac{\partial^3 f(\eta,\hbar,\gamma,q)}{\partial\eta^3} - f(\eta,\hbar,\gamma,q)\frac{\partial^2 f(\eta,\hbar,\gamma,q)}{\partial\eta^2}\ , +4\left(1-\left(\frac{\partial f(\eta,\hbar,\gamma,q)}{\partial\eta}\right)^2\right) \tag{17}$$

$$+M\left(1-\frac{\partial f(\eta,\hbar,\gamma,q)}{\partial\eta}\right)\ .$$

When $q = 0$, we have

$$\mathcal{L}[f(\eta,\hbar,\gamma,q) - f_0(\eta)] = 0, \qquad \Rightarrow f(\eta,\hbar,\gamma,0) = f_0(\eta)\ , \qquad \eta\in[0,+\infty), \hbar\neq 0, \gamma>0 \tag{18}$$

and when $q = 1$, we have

$$0 = \aleph[f(\eta,\hbar,\gamma,1)] \ \Rightarrow f(\eta,\hbar,\gamma,1) = f(\eta)\ , \qquad \eta\in[0,+\infty), \hbar\neq 0, \gamma>0 \tag{19}$$

Hence as q varies from 0 to 1, $f(\eta,\hbar,\gamma,q)$ varies from initial solution $f_0(\eta)$ to the exact solution $f(\eta)$.

Here we choose $f_0(\eta)$, the initial guess, such that it satisfies $\mathcal{L}[f_0(\eta)] = 0$ and the boundary conditions (11). We select

$$\mathcal{L}[C_1 + C_2\eta + C_3 e^{-\gamma\eta}] = 0 \tag{20}$$

and

$$f_0(\eta) = \frac{e^{-\gamma\eta}-1}{\gamma(1+\gamma\lambda)} + K_w + \eta\ , \qquad \gamma>0\ . \tag{21}$$

We here assume that the kth-order deformation derivative given by

$$f_0^{[k]}(\eta,\hbar,\gamma) = \left.\frac{\partial^k f(\eta,\hbar,\gamma,q)}{\partial q^k}\right|_{q=0}, \qquad (k \geq 1) \tag{22}$$

exists. By using Eq. (18) and the Taylor's formula, we have

$$f(\eta,\hbar,\gamma,q) = f_0(\eta) + \sum_{k=1}^{+\infty}\left[\frac{f_0^{[k]}(\eta,\hbar,\gamma)}{k!}\right]q^k \ . \tag{23}$$

We here assume that both $\hbar$ and γ are properly chosen in such a way that the series (23) is convergent at $q = 1$. From Eqs. (19) and (21) at $q = 1$, we find the following relationship between known initial solution $f_0(\eta)$ and the unknown solution $f(\eta)$:

$$f(\eta) = f_0(\eta) + \sum_{k=1}^{+\infty}\frac{f_0^{[k]}(\eta,\hbar,\gamma)}{k!} = \sum_{k=0}^{+\infty}\varphi_k(\eta,\hbar,\gamma) \ , \tag{24}$$

where we define

$$\varphi_0(\eta,\hbar,\gamma) = f_0(\eta), \qquad \varphi_k(\eta,\hbar,\gamma) + \frac{f_0^{[k]}(\eta,\hbar,\gamma)}{k!}, \qquad k \geq 1 \ . \tag{25}$$

In order to find the mth -order deformation equation, we first differentiate Eqs. (15) and (16) m times w.r.t. q and then we set $q = 0$, and finally we divide it by $m!$, to obtain

$$\mathcal{L}[\varphi_m - \chi_m\varphi_{m-1}] = \aleph_m(\eta) \ , \qquad m \geq 1, \qquad \eta \in [0,+\infty), \tag{26}$$

with the corresponding boundary conditions

$$\varphi_m(0,\hbar,\gamma) = \varphi_m'(+\infty,\hbar,\gamma) = 0 \ , \varphi_m'(0,\hbar,\gamma) = \lambda\varphi_m''(0,\hbar,\gamma) \ \ m \geq 1, \hbar \neq 0, \gamma > 0, \tag{27}$$

and

$$\aleph_1(\eta) = \hbar\Big[\varphi_0'''(\eta,\hbar,\gamma) - \varphi_0(\eta,\hbar,\gamma)\varphi_0''(\eta,\hbar,\gamma) + 4\Big(1 - \big(\varphi_0'(\eta,\hbar,\gamma)\big)^2\Big) + M\big(1 - \varphi_0'(\eta,\hbar,\gamma)\big)\Big], \tag{28}$$

$$\aleph_m(\eta) = \hbar\left[\varphi_{m-1}'''(\eta,\hbar,\gamma) - \sum_{k=0}^{m-1}\varphi_{m-1-k}(\eta,\hbar,\gamma)\varphi_k''(\eta,\hbar,\gamma) + 4\sum_{k=0}^{m-1}\varphi'_{m-1-k}(\eta,\hbar,\gamma)\varphi_k'(\eta,\hbar,\gamma) + M\big(1 - \varphi_0'(\eta,\hbar,\gamma)\big)\right], \tag{29}$$

where prime denotes the partial derivative w.r.t. η.

Using Eqs. (21) and (28), we can first calculate $\aleph_1(\eta)$ and then by solving linear equation (26) with boundary conditions (27), we can find $\varphi_1(\eta,\hbar,\gamma)$. In the similar manner, we can calculate $2(\eta)$ by using Eq. (29) and then find $\varphi_2(\eta,\hbar,\gamma)$, and so on.

IV. CONVERGENCEOF THE ANALYTICAL SOLUTION

As suggested by Liao [21], the auxiliary parameter $\hbar$ plays a significant role in controlling the convergence and the rate of approximation for the HAM. It is also to be noted that the HAM provides a great deal of flexibility and freedom for choosing appropriate values of $\hbar$ and γ so as to ensure the convergence of the solution, obtained in the form of infinite series, to $f(\eta)$. To choose $\hbar$, Liao introduced the concept of $\hbar$ -curve that gives an admissible range, called convergence region, for the selection of the suitable values of $\hbar$. It is here to be emphasized that there exists a value of parameter γ for every $\hbar$ belonging to the convergence region which is the most suitable value in the sense that it guarantees the fastest convergence of the given series.

In this context, it is worth mentioning that Turkyilmazoglu [34] has also proposed a new and novel way of finding the optimum value of convergence control parameter $\hbar$ to ensure the convergence of the HAM series in a fastest manner. The proposed method constitutes an alternative to the classical $\hbar$ -level curves method (refer Liao [21]) and the squared residual approach proposed by Liao [35] for the determination of optimal value of the convergence control parameter.

In the present analysis, the values of $f''(0)$ for each particular solution of the distinct magnetic parameter M and mass transfer parameter K_w have been found by way of selecting suitable values of $\hbar$ and γ with the help of $\hbar$ -curve given in Fig. 2. These values have been found after appropriate orders of approximations. These values agree well with the corresponding numerical values of Takhar et al. [4] who tackled the problem by shooting method in conjunction with Runge-Kutta fourth order method

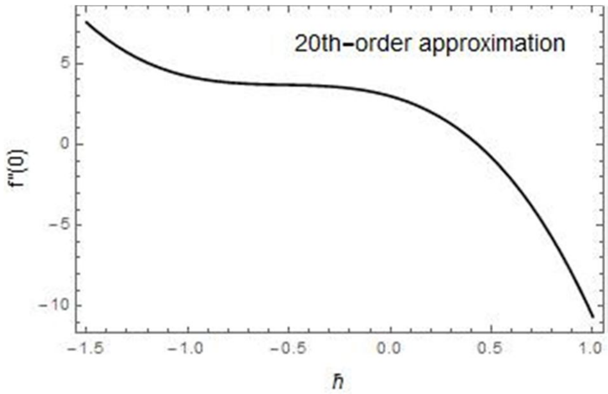

Figure 2: $\hbar$ -curvefor $M = 1, K_w = -2, \lambda = 0$ and $\gamma = 3$

V. Solution For Temperature And Concentration (Eqs. (9)-(13))

The heat and concentration equations can now easily be solved, for the velocity Eq. (8) with boundary conditions (13) already possesses a suitable and accurate solution. It is very much precise to obtain the solution of Eq. (9) by using boundary conditions (12). TABLE I. VALUES OFSKINFRICTION FORDIFFERENTVALUES OFMWHEN $K_w > 0$AND $\lambda = 0$

VI. Results And Discussion

The numerical values of the skin-friction parameter $f''(0)$ for different values of magnetic parameter M and mass transfer parameter K_w (for suction and injection both) have been calculated, and are given in the Tables 1 and 2. From Table 1, it is obvious that the skin-friction parameter $f''(0)$ increases along with the increasing values of the mass suction parameter $K_w(< 0)$ and magnetic parameter M. From the Table 2, it is clear that the skin-friction parameter $f''(0)$ increases along with the increasing values of the magnetic parameter M, but decreases with the increasing values of the mass injection parameter $K_w(> 0)$. The reason for such a behaviour is that both the suction and magnetic parameters reduce the thickness of the momentum boundary layer which results in an increase in skin-friction. The effect of injection is just opposite.

TABLE I:VALUESOF$f''(0)$FORDIFFERENTVALUESOFM WHEN$K_w < 0$AND$\lambda = 0$

M	K_W	Present	Thakaretal. [4]
0	-2	3.5211	3.5182
	-1	2.8517	2.8772
0.5	-2	3.6172	3.6162
	-1	2.9554	3.0231
1	-2	3.7098	3.7124
	-1	3.0542	3.1121

The numerical values of the skin-friction parameter $f''(0)$ for the present case and for the case of Takhar et al. [4] have been given in the Tables 1 and 2 for different values of mass suction/injection and magnetic parameters. These results are in excellent agreement with each other. The mass flux diffusion parameter $(-g'(0))$ have been plotted in Fig. 3 for different values of the magnetic and mass transfer parameters. The graphs thus obtained exhibit excellent agreement with the corresponding graphical results obtained by [4] who visited the problem in the absence of slip parameter ($\lambda = 0$).

From the Fig. 3, it is evident that the heat transfer parameter $(-g'(0))$ decreases for increasing values of the magnetic parameter but increases with increasing values of mass suction ($K_w < 0$). On the other hand, the parameter $(-g'(0))$ decreases with the increasing values of both the magnetic and mass injection parameters. As a result, the suction ($K_w < 0$) reduces the thermal boundary layers, whereas the injection ($K_w > 0$) and the magnetic parameters increase them.

TABLE II: VALUES OF $f''(0)$ FOR DIFFERENT VALUES OF M WHEN $K_w > 0$ AND $\lambda = 0$

M	K_W	Present	Thakaretal. [4]	Rosenhead [1]
0	0	2.2721	2.2728	2.273
	1	1.7861	1.7505	-
	2	1.4167	1.4121	-
0.5	0	2.3827	2.392	-
	1	1.9117	1.973	-
	2	1.5232	1.5529	-
1	0	2.4604	2.4552	-
	1	2.0158	2.0825	-
	2	1.6252	1.6345	-

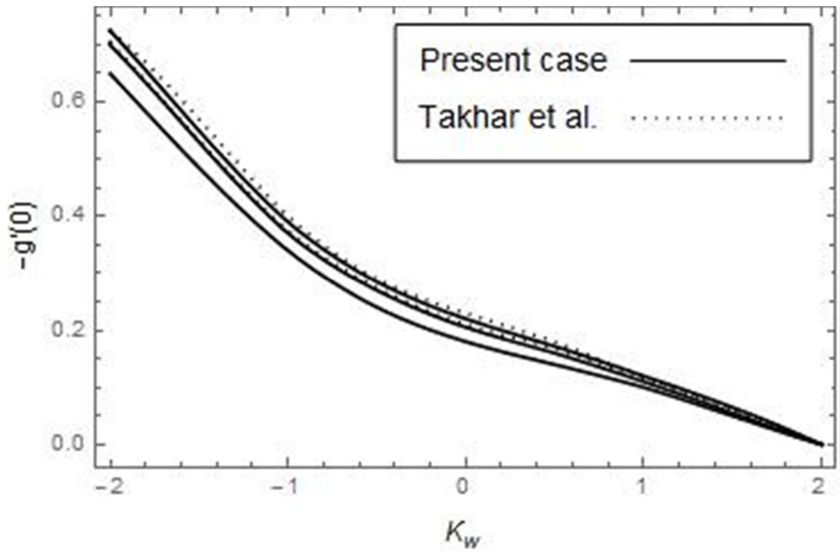

Figure3: Variation of heat transfer and mass diffusion parameters with K_W for M=0,1,2 and Pr=Sc=0.7, λ=0 (Comparison with [4])

It is here to be noted that the mass flux diffusion parameter $(-G'(0))$ is similar to the heat transfer parameter $(-g'(0))$. So, the parameter $(-G'(0))$ will exhibit similar behaviour as $(-g'(0))$ shows in Fig. 3.

For a fixed value of slip parameter, from Fig. 4, it is evident that the velocity profiles exhibit an increasing trend with increasing values of the magnetic parameter (M) in both the cases of suction and injection.

From Fig. 4, it is also clear that the velocity profiles due to suction ($K_w < 0$) are steeper than those due to injection ($K_w > 0$).

As the velocity increases with an increase in magnetic parameter (M), the thickness of the momentum boundary layer also decreases. This happens due to the Lorentz's force arising from the interaction of the magnetic and electric fields during the motion of the electrically conducting fluid.

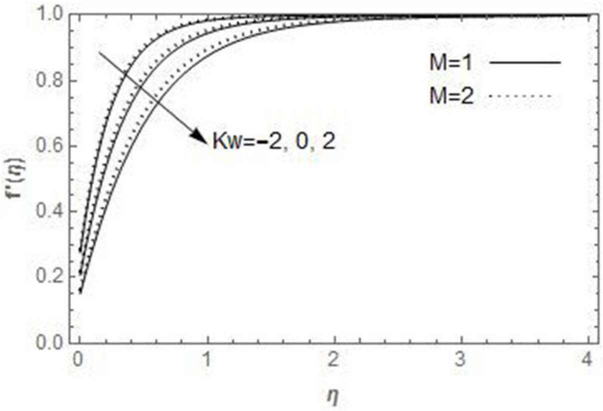

Figure4: Variation of velocity profiles with K_w $(-2,0,2)$, $\lambda=0.1$ for $M=1,2$

In Fig. 5, the nature of temperature profiles with the Prandtl number (Pr) for a fixed value of magnetic parameter (M) has been studied in the absence of the slip effect (i.e. $\lambda = 0$). From the Fig. 5, it is clear that the Prandtl number Pr and hence the Schmidt number Sc, respectively, have significant effects on temperature and concentration profiles. Both Pr and Sc, respectively, increase the temperature and concentration profiles in case of injection. An opposite trend is observed in case of suction.

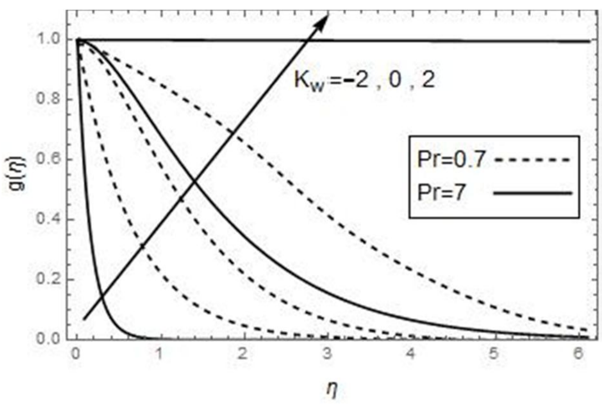

Figure5: Variation of temperature profiles for $M=1$, $Pr(=0.7,7)$, $\lambda=0$, and $K_w(=-2,0,2)$

Fig. 6 depicts the effect of slip parameter λ on velocity profile in presence of magnetic field. It is evident that the velocity profiles exhibit an increasing trend with increasing values of the slip parameter (λ) in both the cases of suction and injection.

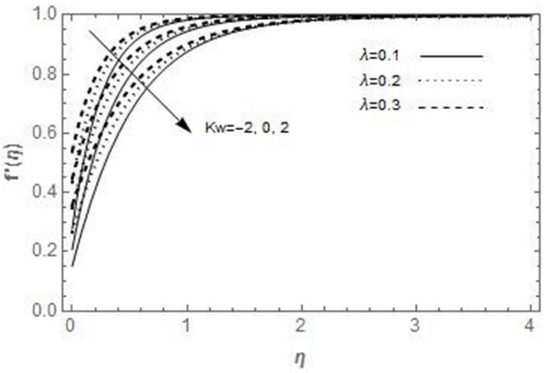

Figure 6: Variation of velocity profiles with K_w $(-2,0,2)$ for $\lambda=0.1,0.2,0.3$ and $M=1$

VII. CONCLUDINGREMARKS

Our results for λ have been found in good agreement with those obtained by Takhar et al. [4] which itself verifies the great potential and validity of the HAM.

The skin-friction increases with increasing magnetic field. The skin-friction is greater for suction parameter $K_w < 0$ as compared to injection parameter $K_w > 0$.

The skin-friction ($f''(0)$), heat transfer parameter ($-g'(0)$) and mass flux diffusion parameter ($-G'(0)$) decrease by injection $K_w > 0$ and increase by suction $K_w < 0$.

The effect of magnetic field on the skin friction, heat transfer and mass flux diffusion is less than the effect of mass transfer.

The temperature and concentration profiles get affected by Prandtl number and Schmidt number, respectively.

To get better results, that is, to get better approximations, HAM can offer us large flexibility and great freedom to choose better auxiliary linear operator ($\mathcal{L}$), nonzero auxiliary parameters ($\hbar$), spatial-scale parameter (γ) for satisfying the rule for solution expression and initial approximations.

With the help of high-speed computers and symbolic computation software like MATHEMATICA, Maple, etc., the HAM might become more powerful and perfect analytic tool to solve rigorous non-linear problems in science and engineering.

ACKNOWLEDGMENT

One of the authors, Chandrakant N. Guled, would like to express his sense of gratitude to Dr. Babasaheb Ambedkar Technological University, Lonere for providing him with TEQIP-II fellowship for pursuing his Ph.D. programme.

REFERENCES

[1] L. Rosenhead, *Laminar boundary layers*, Oxford: Clarendon Press, Oxford, 1963, pp.244–250.

[2] R. C. Ackerberg, "The viscous incompressible flow inside a cone," J. FluidMech., vol. 21(01), pp.47–81, 1965.

[3] S. H. Choi, and H. E. Wilhelm, "Self-similar magnetohydrodynamic diffuser flows with induced magnetic fields," Phys. Fluids, vol. 20, pp. 18–21, 1977.

[4] H. S. Takhar, C. D. Surma Devi, and G. Nath, "MHD flow with heat and mass transfer due to a point sink," Indian J. Pure Appl Math., vol. 17(10), pp. 1242–1247, 1986.

[5] A. T. Eswara, S. Roy, and G. Nath, "Unsteady MHD forced flow due to a point sink," Acta Mech, vol. 145(1-4), pp. 159–172, 2000.

[6] A. T. Eswara, and B. C. Bommaiah, "The effect of variable viscosity on laminar flow due to a point sink," Indian J. Pure Appl. Math., vol.35(6), pp. 811–816, 2004.

[7] M. J. Martin, and I. D. Boyd, "Falkner-Skan flow over a wedge with slip boundary conditions," J. Thermophys. Heat Transfer, vol. 24(2), pp. 263–270, 2010.

[8] D. K. Hutchins, M. H. Harper, and R. L. Felder, "Slip correction measurements for solid spherical particles by modulated dynamic light scattering," Aerosol Sci. Technol., vol. 22(2), pp. 202–218, 1995.

[9] J. C. Harley, Y. Huang, H. H. Bau, and J. N. Zemel, "Gas flow in micro-channels," J. Fluid Mech., vol. 284, pp. 257–274, 1995.

[10] T. J. Mueller, and J. D. DeLaurier, "Aerodynamics of small vehicles," Annu.Rev. Fluid Mech., vol. 35(1), pp. 89–111, 2003.

[11] M. Turkyilmazoglu, "Slip flow and heat transfer over a specific wedge: an exactly solvable Falkner–Skanequation," J. Eng Math., vol. 92(1), pp. 73–81, 2015.

[12] M. Turkyilmazoglu, "Exact multiple solutions for the slip flow and heat transfer in a converging channel," J. Heat Transfer, vol. 137(10), pp. 101–301, 2015.

[13] E. Magyari, "Falkner–Skan flows past moving boundaries: an exactly solvable case," Acta Mech., vol. 203(1-2), pp. 13–21, 2009.

[14] [14] S. J. Liao, *The proposed homotopy analysis technique for the solution of nonlinear problems*, Ph. D. thesis, Shanghai Jiao Tong University, 1992.

[15] S. J. Liao, "Notes on the homotopy analysis method: some definitions and theories," Commun. Nonlinear Sci. Numer. Simulat., vol. 14, pp. 983–997, 2009.

[16] G. Adomian, "Nonlinear stochastic differential equations," J. Math. Anal. Appl., vol. 55, pp. 441–452, 1997.

[17] G. Adomian, "A review of the decomposition method and some recent results for nonlinear equations," Comp.Math. Appl., vol. 21, pp. 101–127, 1991.
[18] R. Rach, "On the Adomian method and comparisons with picards method," J. Math. Anal. Appl., vol. 10, pp. 2346–2356, 1984.
[19] V. M. Soundalgekar, M. Singh, and H. S. Takhar, "MHD free convection past a semi-infinite vertical plate with suction and injection," Nonlinear Anal. Theory Methods Appl., vol. 7(9), pp. 941–944, 1983.
[20] A. M. Lyapunov, *General problem on stability of motion*, Taylor & Francis, London, 1992.
[21] S. J. Liao, *Beyond Perturbation: Introduction to Homotopy AnalysisMethod*. Chapman & Hall/ CRC Press,Boca Raton, 2003.
[22] J. H. He, "Homotopy perturbation technique," Comput. Methods Appl. Mech.Eng., vol. 178, pp. 257–262,1999.
[23] J. H. He, "A coupling method of homotopy technique and perturbation technique for nonlinear problems," Int. J. Non-linear Mech., vol. 35, pp. 37–43, 2000.
[24] S. J. Liao, "Comparison between the homotopy analysis method and the homotopy perturbation method," Appl. Math. Comput., vol. 169, pp. 1186–1194, 2005.
[25] M. Turkyilmazoglu, "Is homotopy perturbation method the traditional Taylor series expansion," Hacet J. Math.Stat., vol. 44(3), 2015.
[26] T. Hayat, and M. Sajid, "Analytic solution for axisymmetric flow and heat transfer of a second grade fluid past a stretching sheet," Int. J. Heat Mass Transfer, vol. 50, pp. 75–84, 2007a.
[27] T. Hayat, Z. Abbas, M. Sajid, and S. Asghar, "The influence of thermal radiation on MHD flow of a second grade fluid," Int. J. Heat Mass Transfer, vol. 50, pp. 931–941, 2007.
[28] T. Hayat, and M. Sajid, "Homotopy analysis of MHD boundary layer flow of an upper-convected maxwell fluid," Int. J. Eng. Sci., vol. 45, pp. 393–401, 2007b.
[29] S. Abbasbandy, "The application of the homotopy analysis method to non- linear equations arising in heat transfer," Phys. Lett. A, vol. 360, pp. 109–113, 2006.
[30] S. Abbasbandy, "Homotopy analysis method for heat radiation equations,"Int. Commun. Heat Mass Transfer, vol.34, pp. 380–387, 2007.
[31] H. S. Takhar, "Hydromagnetic free convection from a flat plate," Indian J. Phys., vol. 45, pp. 289–311, 1971.
[32] H. S. Takhar, and G. Nath, "Similarity solution of unsteady boundary layer equations with a magnetic field," Meccanica, vol. 32(2), pp. 157–163, 1997.
[33] H. Schlichting and K. Gersten, *Boundary layer theory*, Springer, 2000.
[34] M. Turkyilmazoglu, "An effective approach for evaluation of the optimal convergence control parameter in the homotopy analysis method," Filomat, vol. 30(6), pp. 1633–1650, 2016.
[35] S. Liao, *Advances in the homotopy analysis method*. World Scientific, 2014.hed