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Adiga Index for Graphs

Shashirekha. S[1] and Smitha. M[2]
[1,2]Department of Mathematics, JSS Science and Technology University, Mysuru-570 006, India
Email: shashirekhas157@gmail.com, smitham@jssstuniv.in

***Abstract*—In [22] Shashirekha. S et.al, introduced a new variety of topological index called Adiga Index (AI) for a graph G and obtained (AI) for Oxide, Hexagonal networks. Further they conducted QSPR analysis of molecular structures of graphs using AI for physical properties for lower alkanes with AI values and obtained regression models. This article is a continued work of [22], which finds relationship between Adiga Index and Sombor Index for complete and complete bipartite graphs. We also find the relationship between Adiga Index, Elliptic Delta Index and Modified Elliptic Delta Index for Silicate and Honeycomb networks graphs.**

I. INTRODUCTION

We considered graphs that are simple, finite and connected. We have $V(G)$ denoting vertex set and $E(G)$the edge set. Here $d_G(v)$ represents the degree of vertex v, which is nothing but the number of edges incident to the vertex v. A text book by F. Harary [11] is referred to understand standard terminologies and notations. Terminologies which are not standard will be provided when required. A graph index is a numerical value obtained for the graph structure. Quite a lot of graph indices have been defined by using the concept of vertex degree and applied for the study of theoretical chemistry [17]. Graph indices have their applications in various disciplines in science and technology [8][23]. The Banhatti, Gourava, Revan, delta and Zagreb indices are the most degree-based graph indices in the theory of chemical graphs [1][2][3][4][5][6][7][9][10][12][14][15][16][18][19][20]. R. Rajendra et.al [21] introduced Weiner Index and also computed algorithms for Peripheral Wiener Index, Reciprocal Wiener Index and Wiener Index. In [22] Shashirekha. S et.al, introduced a new variety of topological index called Adiga Index (AI) for a graph G and obtained (AI) for Oxide, Hexagonal networks and they have worked on QSPR analysis for lower alkanes using Adiga Index. Inspired by these works we continued our work on Adiga Index (AI) in this article. Here section 2 consists of preliminaries from [22]. In this manuscript we compute the Adiga Index for some special graphs. In we computed Adiga Index for a few graphs. In section 3 we computed Adiga Index for a few chemical structures.

II. PRELIMINARIES

We have the following definition and results from [22]. Adiga Index for a graph *G*, which is a simple graph having n vertices, $v_1, v_2, \ldots, v_n$ is defined by,

$$AI(G) = n^3 + \sum_{uv \in E} max[d(u), d(v)] \qquad (1)$$

Where $d_G(u)$and $d_G(v)$denote the degree of u and v correspondingly for, $i = 1,2,3, \ldots, n$.

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Using the definition of Adiga Index, they have given the following computations

- For Path P_n of *n* vertices,

$$AI(P_n) = n^3 + 2(n-1)$$

- For Cycle $C_n$of n vertices,

$$AI(C_n) = 2|E| = n^3 + 2n$$

- For complete graph $K_n$of n vertices,

$$AI(K_n) = n^3 + \frac{n(n-1)^2}{2}$$

- For fan graph $F_n$of n vertices,

$$AI(F_n) = n^3 + (n-1)^2 + 3(n-2)$$

- For the Star graph S_n of n vertices,

$$AI(S_n) = n^3 + (n-1)^2$$

- For Double Star graph $S_n \times S_n$ on n vertices,

$$AI(S_n \times S_n) = n^3 + n^2$$

- For graph $P_2 \times C_n$ of n vertices,

$$AI(P_2 \times C_n) = n^3 + 9[\frac{n}{2}]$$

- For Triangular ladder graph T_n of n vertices,

$$AI(T_n) = n^3 + 4|E| - 2$$

- For graph $P_n \times C_n$ of n vertices,

$$AI(P_n \times C_n) = n^3 + 4(n+3) + 18$$

III. Computation of Adiga Index for some special and standard graphs.

Example 1. Consider the following Fig 1.1 (G_1)

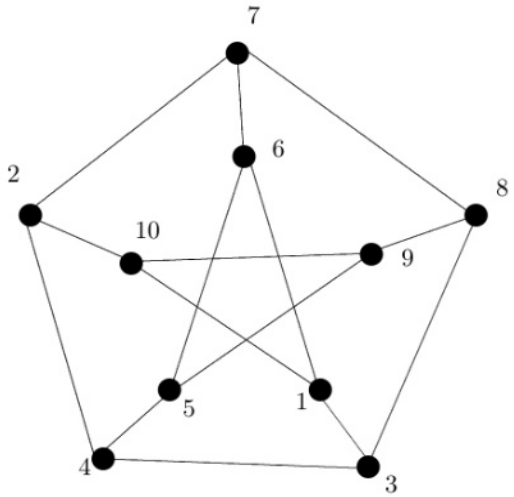

Fig 1(G_1): $AI(G_1) = 1045$

Example 2. Consider the following Fig 2 (G_2)

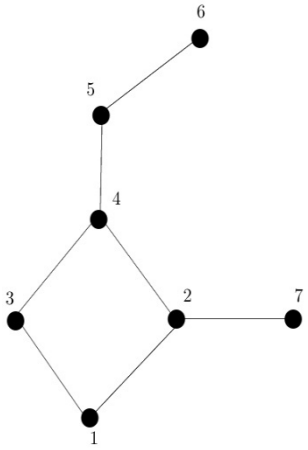

Fig 2: $AI(G_2) = 362$

Example 3. Consider the following Fig 3(G_3)

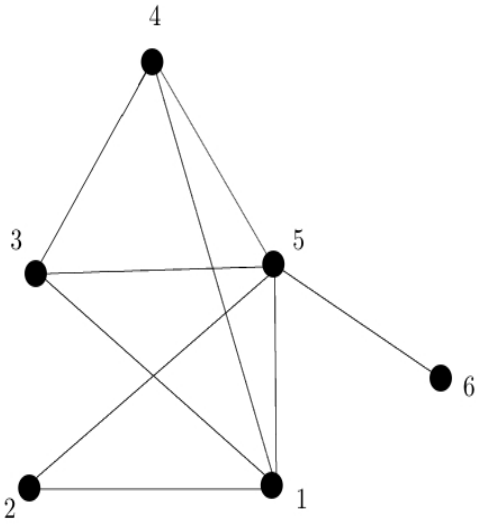

Fig 3: $AI(G_3) = 257$

Theorem 3.1. If G be complete graph with n vertices. Then

$$AI(G) = n^3 + \frac{n(n^2 - 2n + 1)}{2}\varepsilon(G) \quad \forall n$$

Proof. We have

$$AI(G) = n^3 + \sum_{uv \in E} max[d(u), d(v)]$$

$$= n^3 + \frac{n(n-1)^2}{2}(n-1)$$

$$= n^3 + (\sqrt{2}n - \sqrt{2})SO(G)$$

Hence

$$AI(G) > SO(G).$$

and

$$AI(G) = n^3 + (\sqrt{2}n - \sqrt{2})\frac{n(n-1)}{2\sqrt{2}}\,\varepsilon(G)$$

$$AI(G) = n^3 + \frac{n(n^2 - 2n + 1)}{2}\varepsilon(G) \qquad \forall n$$

Theorem 3.2 If G be a complete bipartite graph $K_{n,n}$. Then,

$$AI(G) = \sqrt{2}SO(K_{n,n})$$

and

$$AI(G) = n\varepsilon(G)\ (K_{n,n})$$

Proof. We have

$$AI(G) = n^3 + \sum_{uv \in E} max[d(u), d(v)]$$

$$= n^3 + n^2(n) = 2n^3$$

$$= n\varepsilon\ (K_{n,n})$$

hence

$$AI(G) > \varepsilon\ (K_{n,n}). \qquad \forall n$$

also

$$SO(K_{n,n}) = n^3\sqrt{2}$$

$$AI(G) = \sqrt{2}SO(K_{n,n})$$

hence

$$AI(G) > SO\ (K_{n,n}). \qquad \forall n$$

IV. Some Results Involving Chemical Structure of Certain Networks

A. Adiga Index for Silicate Network

The various forms of silicates are available in nature. Among them we have considered the chemical molecular structure of the compound SiO_4 represents a new interconnection network influenced by chemical molecular structure of a compound SiO_4

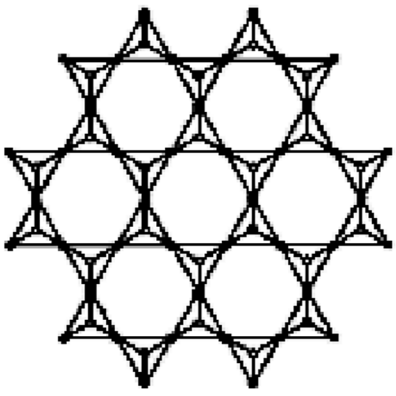

Figure 4: A two-dimensional silicate network

Here, Figure 1.4 gives the graph of silicate network SL_n. The count of nodes in this network is equal to the count of nodes of HC_n + the count of edges of HC_n + 6n boundary nodes. = 6n^2 + (9n^2 − 3n) + 6n = 15n^2 + 3n is the total count of vertices present in the graph.

The count of silicon nodes equals the product of count of nodes of HC_n, and the count of edges of SL_n is $6 \times 6n$ =$36n$. In [13] for graph G we have,

$E_a = \{uv \epsilon E(G) \,|d_G(u) = d_G(v) = 3\}, |E_a| = 6n.$

$E_b = \{uv \epsilon E(G) \,|d_G(u) = 3, d_G(v) = 6\}, |E_b| = 18n^2 + 6n.$

$E_c = \{uv \epsilon E(G) \,|d_G(u) = d_G(v) = 6\}, |E_c| = 18n^2 - 12n.$

Theorem 4.1. If G be a graph of a silicate network SL_n. Then,

$AI(G) = n^3 + 216n^2 - 18n.$

Proof. We have,

$$AI(G) = n^3 + \sum_{uv \in E} max[d(u), d(v)]$$

$= n^3 + 3(6n) + 6(18n^2 + 6n) + 6(18n^2 - 12n)$

$= n^3 + 216n^2 - 18n.$

Also, we have the following results for SL_n,

$AI(G) < E\delta(G)$

i.e., $n^3 + 216n^2 - 18n. < \left(90\sqrt{17} + 576\sqrt{2}\right)n^2 + \left(30\sqrt{17} - 372\sqrt{2}\right)n.$

$AI(G) >^m E\delta(G)$

i.e., $n^3 + 216n^2 - 18n. > \frac{6n}{2\sqrt{2}} + \frac{18n^2+6n}{5\sqrt{17}} + \frac{18n^2-12n}{32\sqrt{2}}.$

B. Adiga Index for Honeycomb Network:

Honeycomb networks are a type of graph that are based on hexagonal plane tessellation and are used as multiprocessor interconnection networks. These networks are very useful in both chemistry and computer graphics and are denoted by HC_n.

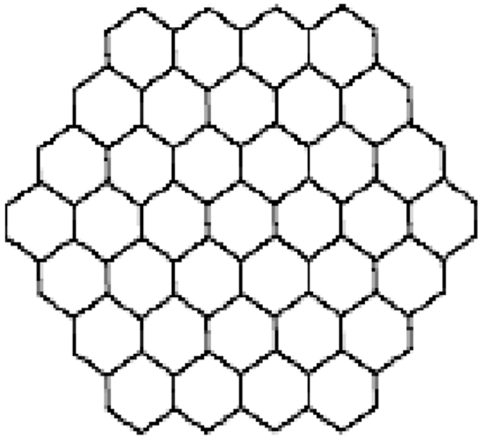

Figure 5: four-dimensional Honeycomb network

Here is Figure 1.5 that gives the graph of honeycomb network HC_n. In [21] V.R. Kulli et.al obtained that graph G *has* $6n^2$ vertices and $9n^2 - 3n$ edges. In G there are three types of edges as follows:

$E_p = \{uv \epsilon E(G) \,|d_G(u) = d_G(v) = 2\}, |E_p| = 6.$

$E_g = \{uv \epsilon E(G) \,|d_G(u) = 2, d_G(v) = 3\}, |E_q| = 12n - 12.$

$E_r = \{uv \epsilon E(G) \,|d_G(u) = d_G(v) = 6\}, |E_r| = 9n^2 - 15n + 6.$

Theorem 4.2. If G be a graph of honeycomb network HC_n. Then,

$AI(G) = n^3 + 27n^2 + 9n - 6$.

Proof. We have,

$$AI(G) = n^3 + \sum_{uv \in E} max[d(u), d(v)]$$

$$= n^3 + 3(12n - 12) + 3(9n^2 - 15n + 6)$$

$$= n^3 + 27n^2 + 9n - 6.$$

Also, we have the following results for HC_n

(i) $AI(G) < E\delta(G)$

i.e.$n^3 + 27n^2 + 9n - 6 < 72\sqrt{2}n^2 + (36\sqrt{5} - 120\sqrt{2})n + 60\sqrt{2} - 36\sqrt{5}$.

(ii) $AI(G) >^m E\delta(G)$

i.e., $n^3 + 27n^2 + 9n - 6 > \frac{6}{2\sqrt{2}} + \frac{12n-12}{3\sqrt{5}} + \frac{9n-15n+6}{8\sqrt{2}}$.

V. CONCLUSION

This article we computed the Adiga Index for a few special graphs. We have computed Adiga Index for complete, complete bipartite, graph with n vertices. We have also obtained AI for Silicate and Honeycomb networks and compared it with elliptic delta index and modified elliptic delta index. We observe that Adiga Index for Silicate and Honeycomb networks is less than
elliptic delta index and greater than modified elliptic delta index.

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