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Reflection of Plane Waves with Two Temperature and Initial Stress in Nonlocal Thermo Elasticity without Energy Dissipation

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***Abstract*— In this paper the governing equations of plane wave in non-local thermoelastic solid half space with initial stress and two temperature is derived. The governing equations of the plane harmonic wave are solved in the x-y plane obtain the three different waves namely thermal wave, P-wave and SV- wave with distinct speeds. Appropriate potential for incident and reflected waves are formulated which satisfies the boundary condition on solid half space. The phase speed and reflection coefficient are computed for a relevant material parameter. The effect of phase speed and reflection coefficient are plotted on the frequency and the effect of reflection coefficient are also plotted at angle of incidence.**

***Index Terms*— Index terms - plane waves, Hydrostatic Initial stress, GN theory of types II and III, Generalized thermos-elasticity, two temperatures, non-local**

I. INTRODUCTION

The basic theory of thermoelasticity as presented in Carlson's paper [1] has been generalized and changed into multiple thermoelastic models known as hyperbolic thermoelasticity presented in {Chandrasekharaiah [2], Hetnarski and Ignazack, [3]}. The thermal waves are modelled in the hyperbolic notation that avoids the physical conundrum of the conventional model's unlimited propagation speed. Green and Naghdi [4-6] put forth three novel thermoelastic theories in the 1990s that were based on entropy equality as opposed to the more common entropy inequality. Each theory has a different set of constitutive assumptions for the heat flux vector. As a result, they were able to develop three ideas that they named type (I,II,III) thermoelasticity. The classical thermoelasticity system is obtained by linearizing the type I theory. The type II theory (which is a limiting case of the type III theory) does not allow for energy dissipation. Wave propagation in thermoelastic materials has numerous applications in science and technology, including atomic physics, industrial engineering, thermal powerplants, underwater constructions, pressure vessels, aircraft, chemical pipes, and metallurgy. Variations in gravity, temperature differences, quenching processes, and other factors can all lead to the development of initial strains in the medium. High initial stresses are expected to be present on Earth. Therefore, it is quite interesting to investigate how these stresses affect mechanical and thermal states of material. The hydrostatic initial load, isotropic linear thermoelasticity was studied by Montanaro [7]. Many authors including Abd-alla and Al-Dawy [8], Sharma et al. [9], and Singh et al. [10], have addressed the problem of plane wave reflection. The existence of novel or altered

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waves can be inferred from investigations of wave propagation in an isotropic generalised thermoelastic solid with extra properties. Experimental seismologists may find this material helpful in estimating the magnitude of earthquakes. Many researchers investigated different types of problems in the theory of thermoelasticity of type III [13-20]. The inelastic behavior of the Earth's material plays an important role in changing the characteristics of seismic waves, in defining seismic source functions Brune [12], and in determining the internal structure of the Earth. The general theory of viscoelasticity describes the linear behavior of both elastic and inelastic materials and provides the basis for describing the attenuation of seismic waves due to in elasticity. Recently, isotropic linear thermoelasticity with hydrostatic starting stress was studied by Montanaro [7]. Since the type II theory does not allow for energy dissipation, it is of importance to us here as a restraining example of type III theory. "Without energy dissipation" is the common term for this idea. Heat conduction in continua can be adequately explained by the thermoelasticity theory without energy dissipation, in accordance with Green and Naghdi [4-6]. Since the development of the theory of initial stress, numerous problems of wave propagation in an initially stressed medium have been discussed by many authors such as Dey, Roy, and Dutta [11]. The format of this paper is as follows: First, two temperature theories and no local are added to the governing equations provided by Montanaro [7] for linear, isotropic, and homogeneous instances in light of the GN theory [6]. Secondly, the expression for the dimensional velocities of three waves is obtained by solving the governing equations analytically for two-dimensional motion in a xy- plane. Finally, both theoretical and numerical calculations are performed for the formulae using reflection amplitude ratios. The numerical results are visually displayed to demonstrate how the reflection amplitude ratios are affected by the thermal boundary condition, the thermoelastic parameter, and the initial stress.

II. Governing Equations

The constitutive relations and field equation for a homogeneous, isotropic non local thermoelastic solid half space with hydrostatic initial stress and two temperatures in the absence of incremental body forces and heat sources are as follows:

$$(1-e^2\Delta^2)\sigma_{ij} = -p\left(\delta_{ij}+\omega_{ij}\right) + 2\mu e_{ij} + \lambda e\delta_{ij} - \frac{\alpha}{k_{\mathrm{T}}}\Theta\delta_{ij} \tag{1}$$

$$h_i = K\frac{\partial\Theta}{\partial x_i} \tag{2}$$

$$U = \frac{\alpha T_o}{\rho_o k_{\mathrm{T}}} e_{kk} + c_e\Theta \tag{3}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j}+u_{j,i}\right) \quad , \quad \omega_{ij} = \frac{1}{2}\left(u_{j,i}-u_{i,j}\right) \tag{4}$$

$$(1-e^2\Delta^2)\rho_o\dot{u}_i = \left(\mu-\frac{p}{2}\right)u_{i,kk} + \left(\lambda+\mu+\frac{p}{2}\right)u_{k,ik} - \frac{\alpha}{k_{\mathrm{T}}}\Theta_{,i}. \tag{5}$$

The heat conduction equation under GN theory (III)

$$K\Theta_{,ii} + k^*\dot{\Theta}_{,ii} = \rho_o c_e T + \gamma\dot{T}_o\dot{u}_{i,i} \tag{6}$$

where $\Theta=(T-a^* T_{,ii})$, T is the temperature above the reference temperature T_0, λ,μ are the Lame parameter, U is the specific internal energy measured from the reference state, u_i are components of displacement vector $\vec{u}$, h_i are components of the heat flux vector, p is the initial pressure, α is the volume coefficient of thermal expansion, k_T is the isothermal compressibility, δ_{ij} is the Kronecker delta, ρ_o, c_e $K(\geq 0)$ and k^* are mass density, specific heat at constant strain, thermal conductivity and material characteristic of the theory, respectively. When $k^*\to 0$ Eq. (7) reduces to the heat conduction equation under the GN theory (II).

III. Formulation Of The Problem

The components of displacement take the following form:

$$u_1 = u(x,y,t), u_2 = v(x,y,t), u_3 = 0 \tag{7}$$

The strain components can be obtained using Equations (4) and (8).

$$e_{11} = \frac{\partial u}{\partial x},\ e_{22}\frac{\partial v}{\partial y},\ e_{33} = 0,\ e_{13} = e_{23} = e_{33} = 0\ ,\ e_{12} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \upsilon = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = u_{i,i} \tag{8}$$

Equations (6) and (7) can be expressed as follows for a two-dimensional solution in a xy-plane:

$$(1 - e^2\Delta^2)\frac{\partial^2 u}{\partial t^2} = \left(\frac{\lambda+2\mu}{\rho_o}\right)\frac{\partial^2 u}{\partial x^2} + \left(\frac{\lambda+2\mu}{\rho_o} - \frac{\mu}{\rho_o} + \frac{p}{2\rho_o}\right)\frac{\partial^2 v}{\partial x\,\partial y} + \left(\frac{\mu}{\rho_o} - \frac{p}{2\rho_o}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\alpha}{k_{\mathrm{T}}\rho_o}\frac{\partial(T - a^*T,_{ii})}{\partial x} \tag{9}$$

$$(1 - e^2\Delta^2)\frac{\partial^2 v}{\partial t^2} = \frac{\lambda+2\mu}{\rho_o}\frac{\partial^2 v}{\partial y^2} + \left(\frac{\lambda+2\mu}{\rho_o} - \frac{\mu}{\rho_o} + \frac{p}{2\rho_o}\right)\frac{\partial^2 u}{\partial x\,\partial y} + \left(\frac{\mu}{\rho_o} - \frac{p}{2\rho_o}\right)\frac{\partial^2 v}{\partial x^2} - \frac{\alpha}{k_{\mathrm{T}}\rho_o}\frac{\partial(T - a^*T,_{ii})}{\partial y} \tag{10}$$

$$\frac{K}{\rho_o}\nabla^2\Theta + \frac{k^*}{\rho_o}\nabla^2\dot{\Theta} - c_e\dot{\Theta} = \frac{\gamma T_o}{\rho_o}\dot{e}, \qquad where, \tag{11}$$

$$W_{\mathrm{T}}^2 = \frac{\lambda + 2\mu}{\rho_o}, W_{\mathrm{S}}^2 = \frac{\mu}{\rho_o}, W_{\mathrm{P}}^2 = \frac{p}{2\rho_o}, \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{12}$$

By using the relations, we introduce the displacement potentials φ and ψ by the relations

$$u = \varphi_{,x} + \psi_{,y}, v = \varphi_{,y} - \psi_{,x} \tag{13}$$

and introduce the subsequent dimensionless variables:

$$\bar{x}_i = \frac{x_i}{V_{\mathrm{T}}\omega_1}, \bar{u}_i = \frac{u_i}{V_{\mathrm{T}}\omega_1}, \bar{\Theta} = \frac{\gamma\Theta}{\lambda+2\mu}, \bar{\varphi} = \frac{\varphi}{(V_{\mathrm{T}}\omega_1)^2}, \bar{\psi} = \frac{\psi}{(V_{\mathrm{T}}\omega_1)^2}, \bar{t} = \frac{t}{\omega_1}, \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\lambda+2\mu}, \omega_1 = \frac{k^*}{\rho_o c_e V_{\mathrm{T}}^2}. \tag{14}$$

Using Eqs. (13) and (14) Eqs. (9)-(11) reduce to (dropping the bar for ease)

$$(1 - e^2\Delta^2)\frac{\partial^2\varphi}{\partial t^2} = \nabla^2\varphi - \beta(T - a^*T,_{ii}) \tag{15}$$

$$(1 - e^2\Delta^2)\frac{\partial^2\psi}{\partial t^2} = (R_{\mathrm{H}} - R_{\mathrm{P}})\nabla^2\psi \tag{16}$$

$$(1 - e^2\Delta^2)\varepsilon_2\nabla^2(T - a^*T,_{ii}) + \varepsilon_3\nabla^2\dot{\Theta} - \dot{\Theta} = \varepsilon_1\nabla^2\dot{\varphi} \tag{17}$$

where e=(1-e^2 Δ^2) ∇^2 φ,R_H=(W_S^2)/(W_T^2),R_P=(W_P^2)/(W_T^2),ε_1=(γ^2 T_o)/(ρ_o^2 c_e V_T^2),β=α/(γk_T) □( ) is usually the thermoelastic coupling factor. ε_2=K/(ρ_o c_e V_T^2) is the characteristic parameter of the GN theory (of type II) and ε_3=k^*/(ρ_o c_e V_T^2 ω_1) is the characteristic parameter of the GN-III theory (of type III). Eqs. (16) and (18) are coupled in φ and Θ, whereas Eq. (17) is uncoupled.

IV. Solution Of The Problem

Equations (16) through (18) can be solved to find the following form for a harmonic wave propagated in the direction where the wave usually lies in the xy plane and makes an angle θ

$$(\phi, T, \psi)(x, y, t) = (\phi, T, \psi)exp(\iota\xi(x\sin\theta + y\cos\theta - \nu\, t) \tag{18}$$

$$T = T_1 exp(\iota\xi(x\sin\theta + y\cos\theta - \nu\, t) \tag{19}$$

Where is the phase speed ξ is the wave number, (sinθ,cosθ) represents the normal wave's projection onto the xy-plane, with φ_1,T_1,ψ_1 being arbitrary constants.

We obtain a system of two homogeneous equations by substituting Eq. (18), (19) into Eqs. (15) and (17).

$$\xi^2(1 - v^2 + e^2)\varphi_1 + \beta\Theta_1 = 0 \tag{20}$$

$$\varepsilon_1 v^2\xi^2\varphi_1 + [-\varepsilon_2(1 - e^2)\varphi_1\xi^2 - \mathrm{i}\varepsilon_3 v\xi - v^2]\Theta_1 = 0 \tag{21}$$

The determinant of the systems (20) and (21) should be zero in order to achieve the non trivial solution of φ_1 and Θ_1. As a result, we obtain

$$(1 - v^2 + e^2)[\mathrm{i}\varepsilon_3 v\omega - v^2] - \beta(\varepsilon_1 v^2 - \varepsilon_2 e^2\Theta_1) = 0, \quad \text{where } \xi v = \omega \tag{22}$$

Eq. (22) is a quadratic equation in v^2 and may be written as

$$(v^2)^2 - \mathrm{P}v^2 + \mathrm{Q} = 0 \qquad where, \tag{23}$$

$$P = 1 + \mathrm{i}\varepsilon_3 + e^2 + \beta\varepsilon_1, \; Q = \varepsilon_2 e^2 \beta \Theta_1 + \mathrm{i}\varepsilon_3 \omega(1 + e^2) \quad (24)$$

The solution of Eq. (23) can be given as $v_1^2 = \frac{P+\sqrt{P^2-4Q}}{2}$, $v_2^2 = \frac{P-\sqrt{P^2-4Q}}{2}$ (25)

$$v_1^2 = \frac{1 + \mathrm{i}\varepsilon_3 + e^2 + \beta\varepsilon_1 + \sqrt{(1 + \mathrm{i}\varepsilon_3 + e^2 + \beta\varepsilon_1)^2 - 4(\varepsilon_2 e^2 \beta \Theta_1 + \mathrm{i}\varepsilon_3 \omega(1 + e^2)}}{2}$$

$$v_2^2 = \frac{1 + \mathrm{i}\varepsilon_3 + e^2 + \beta\varepsilon_1 - \sqrt{(1 + \mathrm{i}\varepsilon_3 + e^2 + \beta\varepsilon_1)^2 - 4(\varepsilon_2 e^2 \beta \Theta_1 + \mathrm{i}\varepsilon_3 \omega(1 + e^2)}}{2}$$

which corresponds to the speeds of the P-wave and thermal wave, respectively. Substituting Eq. (18) into Eq. (16) yields a wave speed of v_3=√(R_H-R_P) for a shear wave. In a two-dimensional motion in the xy-plane of a linear isotropic and homogeneous thermoelastic solid under hydrostatic initial stress and thermal disturbance, there are thus three plane waves: the P-wave, the thermal wave, and the SV wave. The analysis presented above applies to the scenario of a linear isotropic elastic solid. In subsection*{4.1. For incident P-wave}.

V. Reflection On A Stress-Free Surface

We will determine the relations between the reflection amplitude ratios, when P and SV waves incident at the thermally insulated and isothermal stress-free boundary y=0. For an incident (P or SV) wave, the thermal, P and SV-waves will be reflected as shown in fig (1). Accordingly, if the wave normal of the incident wave makes the angle θ with the negative direction of the y-axis, and those of the reflected P, thermal and SV waves make θ_(1,) θ_(2) 〖and θ〗_3 with the same direction.

(a) FOR INCIDENT P-WAVE

The displacement potentials ψ,φ and temperature Θ may be taken in the following form:

$$\varphi = B_o \exp\{\mathrm{i}\xi_1(x\sin\theta_1 + y\cos\theta_1) - \mathrm{i}\omega t\} + A_1 \exp\{\mathrm{i}\xi_1(x\sin\theta_1 - y\cos\theta_1) - \mathrm{i}\omega t\} + A_2 \exp\{\mathrm{i}\xi_2(x\sin\theta_2 - y\cos\theta_2) - \mathrm{i}\omega t\} \quad (26)$$

$$\Theta = \eta_1 \mathrm{B}_0 \exp\{\mathrm{i}\xi_1(x\sin\Theta_1 + y\cos\theta_1) - \mathrm{i}\omega t\} + \eta_1 \mathrm{A}_1 \exp\{\mathrm{i}\xi_1(x\sin\Theta_1 - y\cos\theta_1) - \mathrm{i}\omega t\} + \eta_2 \mathrm{A}_2 \exp\{\mathrm{i}\xi_2(x\sin\theta_2 - y\cos\theta_2) - \mathrm{i}\omega t\} \quad (27)$$

$$\psi = B_1 \exp\{\mathrm{i}\xi_3(x\sin\theta_3 - y\cos\theta_3) - \mathrm{i}\omega t\} \quad (28)$$

$$where, \quad \eta_i = \frac{\xi_i^2(v_i^2-1)}{\beta}, i = 1{,}2 \quad (29)$$

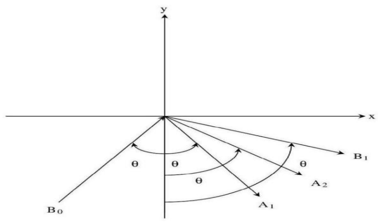


Figure 1: Relation between the incident angle and the reflection angle.

B_0 is the amplitude of incident P-waves, whereas A_1,A_2,B_1 are amplitudes of the reflected P-wave, the thermal wave and the SV-wave, respectively. The ratios of the amplitudes of the reflected waves and the amplitude of the incident wave A_1/B_o ,A_2/B_0 ,B_1/B_0 gives the corresponding reflection coefficients. Also, it may be noted that the angles θ_1,θ_2,θ_3 and the corresponding wave numbers ξ_1,ξ_2,ξ_3 are connected by the relations below

$$\xi_1 \sin\theta_1 = \xi_2 \sin\theta_2 = \xi_3 \sin\theta_3 \quad (30)$$

on the interface y=0 of the medium, relation (30) may also be written as

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3} \quad (31)$$

(b) FOR INCIDENT SV-WAVE

The displacement potentials φ,ψ and temperature Θ occur in the following forms:

$$\varphi = A_2\exp\{i\xi_1(x\sin\theta_2 - y\cos\theta_2) - i\omega t\} + B_1\exp\{i\xi_2(x\sin\theta_3 - y\cos\theta_3) - i\omega t\} \tag{32}$$

$$\Theta = \eta_1 A_2\exp\{i\xi_1(x\sin\theta_2 - y\cos\theta_2) - i\omega t\} + \eta_2 B_1\exp\{i\xi_2(x\sin\theta_3 - y\cos\theta_3) - i\omega t\} \tag{33}$$

$$\psi = B_o\exp\{i\xi_3(x\sin\theta_1 + y\cos\theta_1) - i\omega t\} + A_1\exp\{i\xi_3(x\sin\theta_1 - y\cos\theta_1) - i\omega t\} \tag{34}$$

B_0 is the amplitude of incident SV-waves. The ratios of the amplitudes of the reflected waves and the amplitude of the incident wave A_1/B_0 ,A_2/B_0 ,B_1/B_0 gives the corresponding reflection coefficients. Also, it may be noted that the angles θ_1,θ_2,θ_3 and the corresponding wave numbers ξ_1,ξ_2,ξ_3 are linked by the relations below

$$\xi_3\sin\theta_1 = \xi_1\sin\theta_2 = \xi_2\sin\theta_3 \tag{35}$$

on the interface y=0 of the medium, relation (35) may also be written as

$$\frac{\sin\theta_1}{v_3} = \frac{\sin\theta_2}{v_1} = \frac{\sin\theta_3}{v_2} \tag{36}$$

VI. BOUNDARY CONDITIONS

For the stress-free surface y=0, the boundary conditions are given by

$$(1-e^2\Delta^2)\sigma_{yy} = -p,\quad (1-e^2\Delta^2)\sigma_{yx} = 0,\quad \frac{\partial\Theta}{\partial y} + h\Theta = 0,\qquad at\ y = 0 \tag{37}$$

where h→0 corresponds to the thermal insulated boundary (Singh et al.) [8] and h→∞ to the isothermal boundary conditions. The dimensionless boundary conditions can be written as follows:

$$(1-e^2\Delta^2)\sigma_{yy} = -R_{\mathrm{P}},\qquad (1-e^2\Delta^2)\sigma_{yx} = 0,\qquad \frac{\partial\Theta}{\partial y} + hV_{\mathrm{T}}\omega_1\Theta = 0,\ at\qquad y = 0 \tag{38}$$

The dimensionless variables σ_yy,σ_yx are given below

$$(1-e^2\Delta^2)\sigma_{yy} = -R_{\mathrm{P}} + 2R_{\mathrm{H}}\frac{\partial v}{\partial y} + (1-2R_{\mathrm{H}})e - \beta\Theta \tag{39}$$

$$(1-e^2\Delta^2)\sigma_{yx} = 2(R_{\mathrm{H}} - R_{\mathrm{P}})\frac{\partial u}{\partial y} + 2(R_{\mathrm{H}} + R_{\mathrm{P}})\frac{\partial v}{\partial x} \tag{40}$$

VII. EXPRESSIONS FOR THE REFLECTION AMPLITUDE RATIOS FOR THE INCIDENT P-WAVE

The following relations can be obtained by applying boundary conditions (37) and Eqs. (26)–(28).

$$\frac{A_1}{B_o}a_{11} + \frac{A_2}{B_o}a_{12} + \frac{B_1}{B_o}a_{13} = b_1, \tag{41}$$

$$\frac{A_1}{B_o}a_{21} + \frac{A_2}{B_o}a_{22} + \frac{B_1}{B_o}a_{23} = b_2, \tag{42}$$

$$\frac{A_1}{B_o}a_{31} + \frac{A_2}{B_o}a_{32} + \frac{B_1}{B_o}a_{33} = b_3, \tag{43}$$

Where , Where , $a_{1i} = -\left(\frac{2R_{\mathrm{H}}\omega^2\cos^2\theta_i}{v_i^2} + \frac{(1-2R_{\mathrm{H}})\omega^2}{v_i^2} - \beta\eta_i\left(1 + a^*\frac{\omega^2}{v_i^2}\right)\right)(i = 1,2),\ a_{13} = \frac{2R_{\mathrm{H}}\omega^2\sin 2\theta_3}{v_3^2},$

$a_{2i} = \frac{R_{\mathrm{H}}\omega^2\sin 2\theta_i}{v_i^2}(i = 1,2), a_{23} = (R_{\mathrm{P}} - R_{\mathrm{H}}\cos 2\theta_3)\frac{\omega^2}{v_3^2},$

$a_{3i} = \left(\frac{i\omega\cos\theta_i}{v_i} + \zeta - a^*\frac{\omega^2}{v_i^2}\right)\eta_i(i = 1,2), a_{33} = 0,$

$b_1 = a_{11}, b_2 = a_{21}, b_3 = \left(\frac{i\omega(\cos\theta_1 - a^*)}{v_1} + \zeta\right)\eta_1,$ where $\zeta = hV_{\mathrm{T}}\omega_1$

The solution of this system for the reflection amplitude ratios, $z1 = \frac{A_1}{B_o} = \frac{P_1}{Q_1}, z2 = \frac{A_2}{B_o} = \frac{P_2}{Q_1}, z3 = \frac{B_1}{B_o} = \frac{P_3}{Q_1}$ (44)

(b) FOR THE INCIDENT SV-WAVE

The following relations can be obtained by applying boundary conditions (38) and Eqs. (32)– (34).

$$\frac{A_2}{B_o}c_{11} + \frac{B_1}{B_o}c_{12} + \frac{A_1}{B_o}c_{13} = d_1, \quad (45)$$

$$\frac{A_2}{B_o}c_{21} + \frac{B_1}{B_o}c_{22} + \frac{A_1}{B_o}c_{23} = d_2, \quad (46)$$

$$\frac{A_2}{B_o}c_{31} + \frac{B_1}{B_o}c_{32} + \frac{A_1}{B_o}c_{33} = d_3, \quad (47)$$

where,

$$c_{1i} = \left(\frac{2R_{\mathrm{H}}\omega^2\cos^2\theta_{i+1}}{v_i^2} - \beta\eta_i\left(1 + a^*\frac{\omega^2}{v_i^2}\right) + \frac{(1-2R_{\mathrm{H}})\omega^2}{v_i^2}\right)(i = 1,2), c_{13} = -2R_H$$

$$c_{2i} = \frac{R_{\mathrm{H}}\omega^2\sin 2\theta_{i+1}}{v_i^2}(i = 1,2), c_{23} = (R_{\mathrm{P}} - R_{\mathrm{H}}\cos 2\theta_1)\frac{\omega^2}{v_3^2},$$

$$c_{3i} = \left(\frac{\mathrm{i}\omega\cos\theta_{i+1}}{v_i} - a^* i\omega(1-\omega_1) + \zeta\right)\eta_i(i = 1,2), c_{33} = 0$$

$$d_1 = -c_{13}, d_2 = -c_{23}, d_3 = 0$$

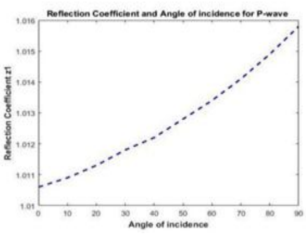
The solution of this system for the reflection amplitude ratios $z1 = \frac{A_2}{B_o} = \frac{R_1}{Q_2}, z2 = \frac{B_1}{B_o} = \frac{R_2}{Q_2}, z3 = \frac{A_1}{B_o} = \frac{R_3}{Q_2}$ (48)

VIII. NUMERICAL RESULTS

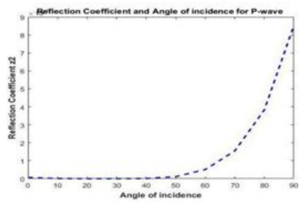
The general Lame's constants are $\lambda = \frac{E\sigma}{\eta(1+\sigma)(1-2\sigma)}, \quad u = \frac{E}{2\eta(1+\sigma)}$

E is Young's modulus, where η is the initial stress parameter and σ is the Poisson ratio. $\eta = 1$ corresponds to the isotropic elastic medium. The numerical calculations are performed using the following dimensionless parameters for a generalized thermoelastic solid under hydrostatic stress.

$E = 36.9 \times 10^{10}\mathrm{Nm}^{-2}, \sigma = 0.2, \omega = 5.0, \varepsilon_2 = 0.0198, \varepsilon_3 = 0.05, \beta = 1.0$



2(a)



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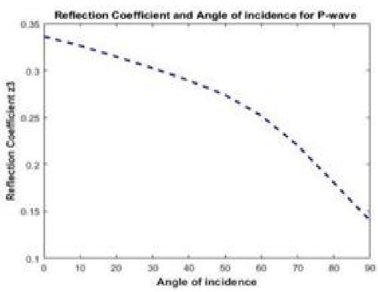
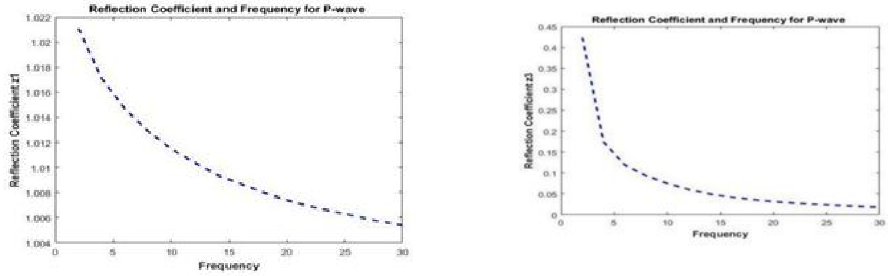


2(c)

Figure 2: Reflection coefficient with respect to angle of incidence

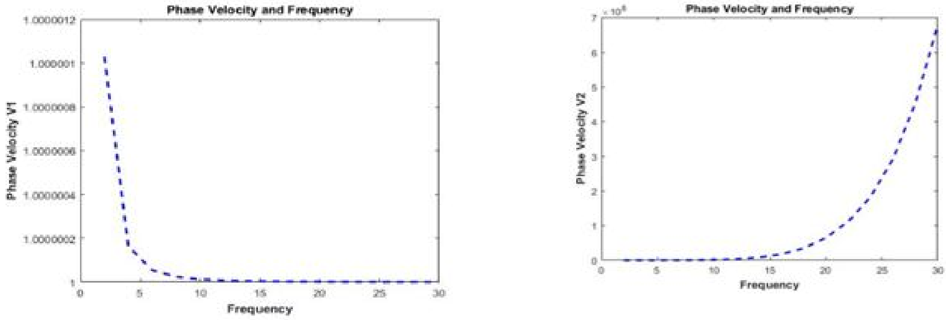
Fig. 2 gives the reflection coefficient z1,z2,z3 with respect to angle of incidence for the incident P-wave. From Fig. 2(a) we can see that the reflection coefficient zl has a constant increase along the angle of incidence, whereas in figure 2(b) reflection coefficient z2 has constant path between 0^0≤θ≤〖50〗^∘, then there is rapid increase in z2 with increase in the angel of incidence as shown in the graph, we can see that z3 shows decrease as we increase the angle of incidence.



3(a) 3(b)

Figure 3: Reflection coefficient with respect frequency

As in fig. 4 we have plotted a graph between the reflection coefficient z1,z3 with respect to the frequency of p-wave, in which we can see that for the reflection coefficient z1 of p wave decreases with respect to the increase in the frequency in fig 2(a), the reflection coefficient z3 shows a rapid decrease as we increase the value of frequency between 0 -5 then z3 decreases slowly with respect to frequency.



4(a) 4(b)

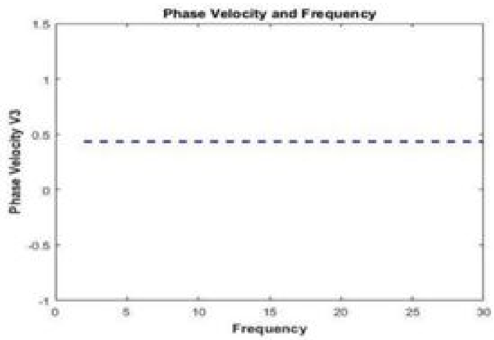


Figure 4: Phase velocity with respect to frequency

Fig. 4 represents the phase velocity in accordance to the frequency. In fig 4(a) phase velocity decrease when we increase the frequency till frequencies 0 to 5 then it remains constant throughout with respect to frequency, for phase velocity V2 the graph shows constant values between frequencies 0 to 15 and then increases slowly in fig. 4(b) whereas for V3 the value remains constant with respect to phase velocity V3 and frequency applied throughout in fig.4(c).

IX. CONCLUSION

In this paper we studied Reflection of plane waves with two temperature and initial stress in nonlocal thermos-elasticity without energy dissipation by using the governing equations for half space under Green and Naghdi theory of kinds II and III to obtain the fundamental governing equation for isotropic and homogeneous generalized thermos-elastic half space under hydro-static initial stress using two temperatures. The dimensionless velocities in the x-y plane are obtained by analytically solving these governing equations. We came to the determination that, the reflection coefficients z1,z2,z3 for p-wave depends on the angle of incidence and shows different results for different angle of incidence for reflection coefficients. Reflection coefficient of p wave shoes different results for different frequencies. Phase velocity V_1, V_2, V_3 of P-wave changes with respect to the frequencies applied.

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