Grenze International Journal of Engineering and Technology, June Issue

Secure PMU based Low Frequency Oscillation Identification SMIB System

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***Abstract*— Modern Power systems are equipped with data source devices such as Phasor measuremet unit s and Phasor data concentrators and securing the synchronized measurement data of power system is challenging task. Technological development in the data driven techniques has reduced the complexity involved in the processing and analysis the PMU data. However, the with the data secuiry the analysis of power system behaviour has posed challenged to data and power engineer. The paper addressess the performance of various data driven and ML techniques to analyse and identify the low frequency oscillation eigenvalues under the secure environment. The synthetic data is prepared for SMIB system under MATLAB/ Simulink environment.**

***Index Terms*— first term, second term, third term, fourth term, fifth term and sixth term.**

I. INTRODUCTION

Modern Cyber Physical Infrastructure of Power system is installed with smart devices and monitoring systems produces enormous data. The processing of it poses a new challenge for the power and data engineering's for the adequate decision making and detection of anomalies [9] [10][11] There is time tradeoff between the implementation of data security in various stages of smart power system and time of execution. Thus, it is crucial to have the right methods for identifying anomalies The cyber attacks is the major drawback of the cyber physical power system which needs to be monitored and mitigated spontaneously. Researchers are currently focusing on stability concerns from the perspective of cyber physical power systems.

Realtime estimation of low frequency oscillations in the power system is major concern in the wide area monitoring system. Many algorithms are suggested for the Realtime identification using online and offline studies of Identification of oscillatory frequencies and damping factor of the such oscillations under small disturbances in the power system. Data driven techniques are discussed in the [1][2] suggest for the scope for improvement on identification accuracy and computational complexity. The identification accuracy in the Prony based approaches are poor during noisy environment and missing of samples. The accuracy of prediction depends on additional filter and interpolation techniques, however the Prony based methods gives poor prediction during noisy environments [3][4]. Empirical Mode decomposition and Dynamic mode decompositions are widely used for the identification of LFOs for the data from PMUs and PDC[6-13] However, they are prone to the noise and will lead into mis identification.

The paper presents the comparative analysis of VMD based identification approach with Machine learning based Techniques.

Grenze ID: 01.GIJET.12.2.511

The effectiveness of the approaches is validated comparing the percentage error in damping factor and frequency of oscillation of the mode under study.
The energy computation is carried out for the predicted signal with original PMU signals, it has been observed that, the secured data of PMU after being encrypted has no impact on the identification of eigenvalues of the system.
Section II describes the various techniques used in the execution of the results. Section III describes the results and section IV is conclusion.
Section II:

A. Prony Approximation

Consider the signals extracted from the PMU data of multimachine power system which is represented as x(0),x(1),………x(N-1) are approximated by the exponential components

$$\hat{x}(n) = \sum_{k=1}^{q} a_k z_k^n \quad \text{for} \quad (n = 0,1, \ldots \ldots N-1) \qquad (1)$$

Where $a_k = A_k e^{j\theta_k}$
and $z_k = e^{[(\sigma_k + 2\pi f_k)\Delta t]}$
Where q is the order, Δt is sampling interval A_k , θ_k, σ_k and f_k are amplitude, phase ,attenuation factor and frequency. The objective of the Prony algorithm is to calculated the parameter a_k and z_k.Here, $\hat{x}(n)$ is $x(n)$'s approximated value and error can be calculated as

$$er(n) = x(n) - \hat{x}(n) \quad \text{for} \quad n = 0,1, \ldots \ldots . N-1 \qquad (2)$$

The constant coefficient linear differential equations can be obtained as

$$\hat{x}(n) = -\sum_{k=1}^{q} b_k \hat{x}(n-k) \qquad n = q, \ldots \ldots N-1 \qquad (3)$$

The points on the z-plane z_k are the solutions of the equation

$$1 + \sum_{k=1}^{q} b_k z^{-k} = 0 \qquad (4)$$

The error equation can be obtained from equation (3) and (4) is

$$x(n) = -\sum_{k=1}^{q} b_k \hat{x}(n-k) + \boldsymbol{\Phi}(n) \quad n = q, \ldots \ldots N-1 \qquad (5)$$

Where $\boldsymbol{\Phi}(n)$ satisfies the following equation

$$\boldsymbol{\Phi}(n) = -\sum_{k=0}^{p} b_k er(n-k) \qquad (6)$$

Using Least square algorithm the error can be minimized as
$\sum_{n=q}^{N-1} |\boldsymbol{\Phi}(n)|^2$ the coefficients $b1, b2, \ldots \ldots . b_q$ is obtained with the following equation

$$\begin{bmatrix} M(1,0) & M(1,1) \ldots \ldots & M(1,q) \\ M(2,0) & M(2,1) \ldots \ldots & M(2,q) \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} 1 \\ b_1 \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix} \qquad (7)$$

Where $M(i,j) = \sum_{n=q}^{N-1} x(n-j)x(n-k) \ \ where \ \ k,j = 0,1,2, \ldots . q$
The following equation can be obtained from the equation (1)

$$\begin{bmatrix} z_1^0 & \ldots & z_q^0 \\ \vdots & \vdots & \vdots \\ z_1^{N-1} & \ldots & z_q^{N-1} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_q \end{bmatrix} = \begin{bmatrix} \hat{x}(0) \\ \vdots \\ \hat{x}(N-1) \end{bmatrix} \qquad (8)$$

Where $\hat{x}(n) = \begin{cases} x(n) & 0 \le n \le q \\ -\sum_{k=1}^{q} b_k \hat{x}(n-k) & q+1 \le n \le N-1 \end{cases}$

The computation of coefficients a_k is carried out using procedures of Prony algorithm from equations(5) and (8).

B. Variational Mode Decomposition

VMD decomposes a composite signal into y(t)into number of Intrinsic Mode Function (IMFs) i_n(t)[33]

$$y(t)\sum_{n=1}^{K} i_n(t)$$

The features of these IMFs are characterised by i. Each modes $i_n(t)$ is the amplitude and frequency modulated signals of the form $i_n(t) = A_n(t)\cos(\varphi n(t))$ where $\varphi_n(t)$ is the phase of the mode and An(t) is the envelop. The envelopes of the modes exhibit slow variations and positive values The mode frequency of each IMFs is non-decreasing and concentrated around the central value f_n.

C. Singular Spectrum Analysis (SSA)

Let the observed time series be:x = $\{x_1, x_2, x_3, \ldots, x_n\}$, where N is the total number of samples.
x(t) = x_trend(t) + x_osc(t) + x_noise(t)
Choose window length: L such that 2 ≤ L ≤ N/2 and K = N − L + 1

$$2 \le L \le \frac{N}{2}, \quad K = N - L + 1$$

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & \cdots & x_K \\ x_2 & x_3 & \cdots & x_{K+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_L & x_{L+1} & \cdots & x_N \end{bmatrix}$$

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$$

Then Computes the Single value decomposition (SVD) of the Matrix X.[] Grouping method has been carried out once the eigenvalues are obtained[]. Also

2) Step 2: Singular Value Decomposition:

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \tag{10}$$

$$\mathbf{\Sigma} = \mathrm{diag}(\sigma_1, \ldots, \sigma_d)$$

Each component:

$$\mathbf{X}_i = \sigma_i \mathbf{U}_i \mathbf{V}_i^T \tag{11}$$

3) Step 3: Grouping:

$$\mathbf{X}^{(g)} = \sum_{i \in I_g} \mathbf{X}_i \tag{12}$$

4) Step 4: Diagonal Averaging: For $1 \le k < L$:

$$\tilde{x}_k = \frac{1}{k}\sum_{i=1}^{k} y_{i,k-i+1} \tag{13}$$

For $L \le k \le K$:

$$\tilde{x}_k = \frac{1}{L}\sum_{i=1}^{L} y_{i,k-i+1} \tag{14}$$

For $K < k \le N$:

$$\tilde{x}_k = \frac{1}{N-k+1}\sum^{L} y_{i,k-i+1} \tag{15}$$

Result:

Comparison between Prony and Vmd

Prony analysis and Variational Mode Decomposition (VMD) are both used for oscillation mode identification, but their performance differs significantly under noisy and secured PMU data conditions. Prony analysis models the signal as a sum of damped exponentials and directly estimates system poles. However, its accuracy deteriorates in the presence of noise, missing samples, or encrypted data, because it relies on precise time-domain fitting. Small disturbances in the signal can lead to large deviations in estimated eigenvalues.

In contrast, VMD first decomposes the signal into band-limited intrinsic mode functions (IMFs) before modal estimation. This preprocessing step isolates oscillatory components and suppresses noise, resulting in more reliable eigenvalue estimation. Figure 1 shows the comparison of both Porny and VMD approximation for the PMU signal from SMIB system. It can be observed that, the prony has poor approaximation compared to VMD.

Also it can be seen from the figure 2 that, Prony reaches to saturation after 10 order with RMSE of 0.4652. Therefore the Prony analysis is going to be poor identification approach for PMU signal. Eigenvalues estimated in the Table 1 shows that error of detection is very huge which is not acceptable. Figure 4 depicts the approximation error usinf VMD approach and Table 2 indicates the performance of VMD outsmart the Prony based approaches. The energy of reconstructed signal using VMD approximation is almost equal to the original signal. It can also inferred from the Table 2 that, VMD gives best approximation.

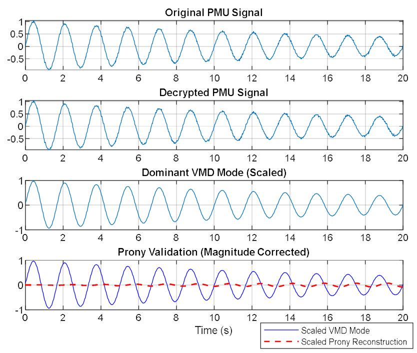


Figure 1: Approximation of signal using Prony and VMD

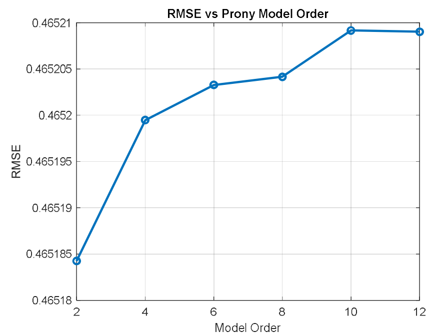


Figure 2: Order of Prony

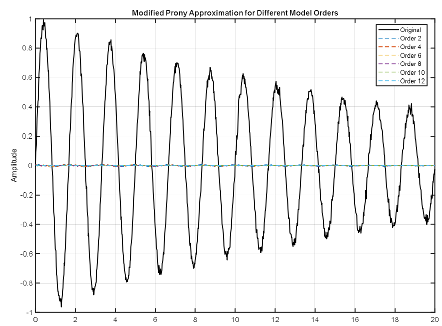
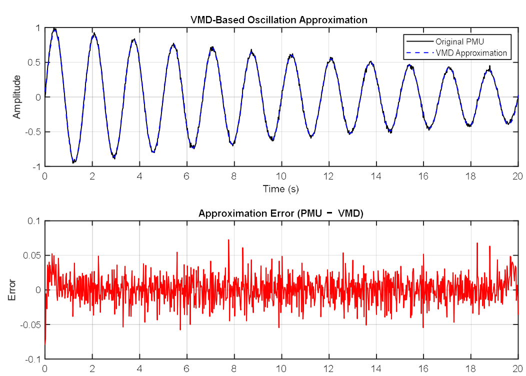


Figure 3: Approximation of signal using Prony

TABLE I: EIGENVALUE ESTIMATION

Method	Real Part	Imag Part	Freq Err %	Damp Err %
True System	-0.0500	3.7699	-	-
Prony Only (Raw)	-0.5662	0.0000	100.000	1032.447
Prony Only (Enc)	-0.5661	0.0000	100.000	1032.239



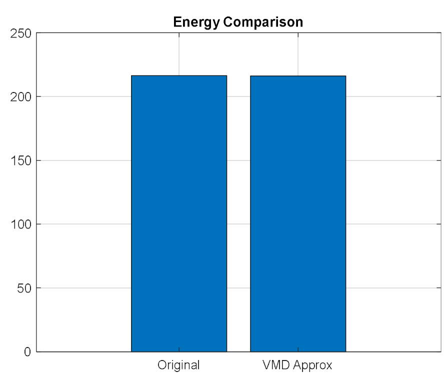


Figure 4: VMD approximation

Figure 5: Energy Comparison

TABLE II: VMD APPROXIMATION METRICS

Original Signal Energy	216.4541
VMD Approx Energy	216.0756
Energy Error (%)	0.175 %
RMSE (VMD Approx)	0.019445

TABLE III: VMD EIGENVALUE IDENTIFICATION

True Eigenvalue	0.0500 +3.7699j
Estimated Eigenvalue	0.0474 +3.7661j
Frequency Error (%)	0.100 %
Damping Error (%)	5.221 %

the percentage errors in the detection of frequency and damping of Low frequency Oscillation.
Comparison of VMD with SSA based Oscillation Monitoring system: Encrypted PMU signal is decrypted using relevant algorithms before the identification method. Figure 6 shows the flowchart of process from capturing of the signal to the identification of eigenvalues. It can be seen that from the figure 8.

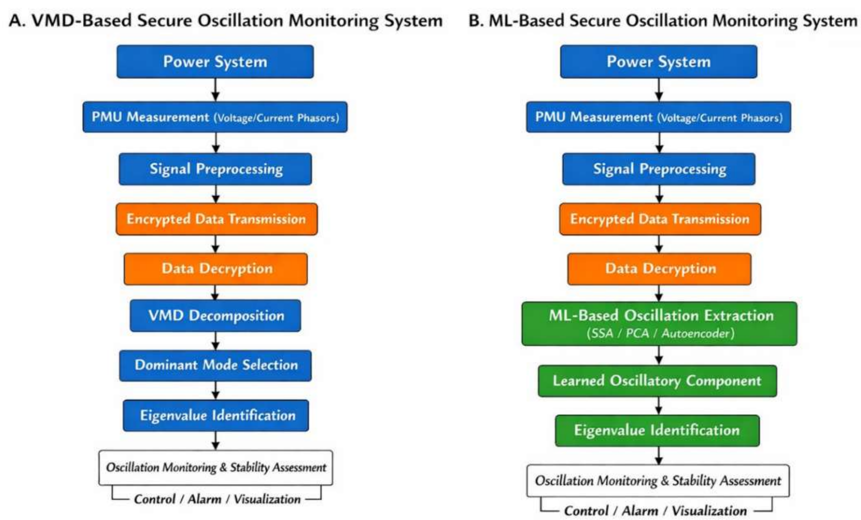


Figure 6: Flowchart for the comparison of VMD with SSA

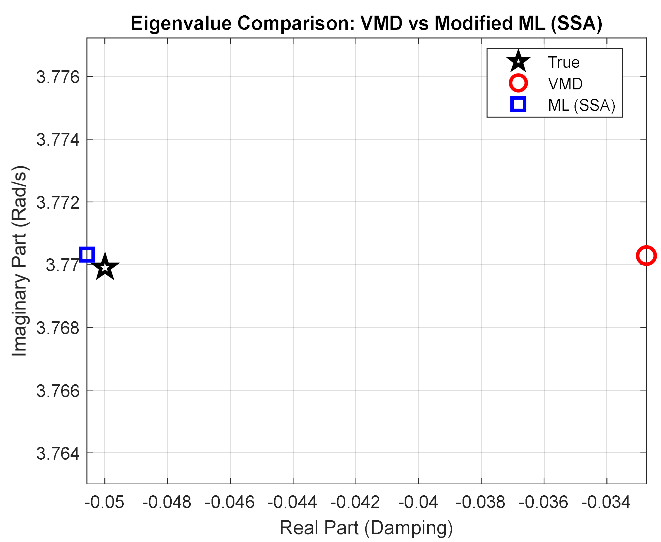


Figure 7: Eigenvalue compared with original value

TABLE IV: EIGENVALUES

Method	Real Part	Imag Part	Freq Err %	Damp Err %
True	0.0500	3.7699	-	-
VMD	0.0474	3.7661	0.016	5.21
ML (SSA)	0.0503	3.7694	0.014	0.01

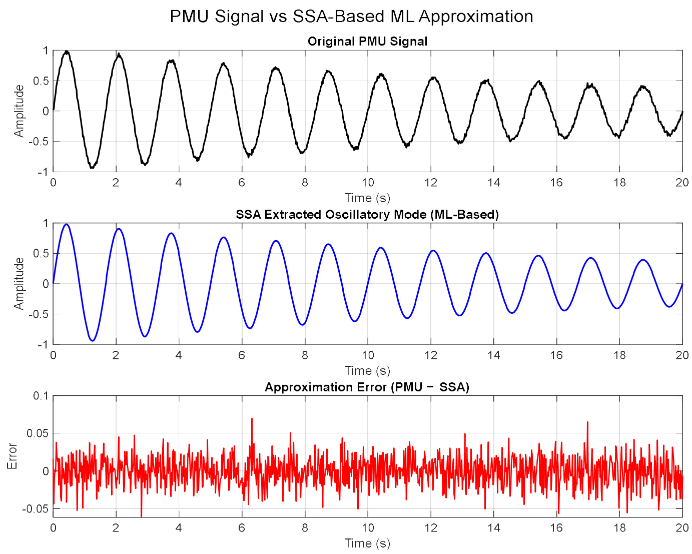


Figure 8: SSA approximation

Singular Spectrum Analysis (SSA) and Variational Mode Decomposition (VMD) both enhance low-frequency oscillation identification from secured PMU data by separating oscillatory components from noise. SSA performs decomposition in the time domain using singular value separation and does not require prior knowledge of the number of modes, making it computationally efficient and suitable for adaptive monitoring. However, its ability to distinguish closely spaced oscillatory modes can be limited. VMD, in contrast, operates in the frequency domain and decomposes the signal into band-limited intrinsic mode functions centered around adaptive frequencies. This enables better mode separation and stronger noise suppression. As a result, VMD generally achieves lower frequency and damping estimation errors and provides more accurate waveform reconstruction than SSA. While SSA remains a fast and robust alternative for real-time applications, VMD is preferred when higher precision in eigenvalue estimation is required under noisy or encrypted PMU conditions.

II. CONCLUSION

This work presented a secure PMU-based framework for low-frequency oscillation identification in an SMIB system using Prony analysis, Variational Mode Decomposition (VMD), and Singular Spectrum Analysis (SSA). The study demonstrated that classical Prony analysis is highly sensitive to noise and encrypted data, leading to large errors in eigenvalue estimation. In contrast, both VMD and SSA significantly improved oscillatory mode extraction by effectively separating noise and preserving signal dynamics. Among the two, VMD provided the highest accuracy in estimating oscillation frequency and damping due to its adaptive, band-limited decomposition capability, while SSA offered a computationally efficient and model-free alternative suitable for real-time monitoring. The results confirm that secure data processing does not significantly degrade oscillation identification performance when robust signal decomposition techniques are employed. Future work may focus on real-time hardware implementation and extending the approach to multi-machine power systems.

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