

A Radar Target Tracking System for Spiral Paths based on Kalman Filter

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Abstract— This paper focuses on linear Kalman Filter for Radar Target Tracking. Kalman filter uses recursive computations to attain a solution of discrete linear filter. Being an adaptive filter, Kalman filter analysis the relation between its estimated value and measured value, through a feedback loop and tries to attain the result after minimising the noises in the measured value. This research examines how measurement noise affects tracking performance of Radars in three dimensions in terms of relative prediction error at various SNR.

Index Terms— Kalman Filter, Radar target tracking, measurement noise, relative error, signal to noise ratio

I. INTRODUCTION

In modern radar systems, accurate and robust target tracking is crucial for applications ranging from air traffic control to military surveillance [1]. The Radar Target Tracking System based on the Kalman Filter stands as a sophisticated solution to address the challenges posed by noisy measurements and dynamic target motion. The Kalman Filter, an optimal recursive algorithm, plays a pivotal role in estimating and updating the state of a moving target, making it a cornerstone technology in radar signal processing [2].

This system operates on a well-defined model comprising a state vector representing the target's position and velocity, coupled with a measurement vector reflecting radar-derived data. The dynamic evolution of the target state over time is encapsulated in a system dynamics model, while the measurement model characterizes the relationship between the state vector and the noisy radar measurements [4]. Subsequently, the Kalman Filter alternates between prediction and update steps. The prediction step utilizes the system dynamics model to forecast the next state, updating the error covariance matrix accordingly. The update step incorporates the actual radar measurements, computing the Kalman Gain to adjust the state estimate based on the comparison between the predicted and measured values. This iterative process ensures a continuous refinement of the target state estimate as new measurements become available [5].

The elegance of the Kalman Filter lies in its ability to balance the information derived from both prediction and measurement sources, effectively mitigating the impact of measurement noise and uncertainties in the system. This results in a reliable and accurate estimation of the target's state, making the Radar Target Tracking System with Kalman Filter a robust solution for real-world radar applications.

By seamlessly integrating advanced mathematical principles with radar technology, this system not only enhances tracking precision but also contributes significantly to the overall efficiency and reliability of radar systems across diverse domains. The Kalman Filter-based Radar Target Tracking System stands as a testament to the continual advancement of signal processing methodologies, ensuring the continued evolution of radar technology in the pursuit of heightened situational awareness and tracking accuracy.

II. THEORY

Kalman Filter is an optimal recursive estimation algorithm that uses indirect and uncertain measurements to estimate the state of a dynamic system. It is ideal for linear systems that have Gaussian noise. The term "Filter" is used because it eliminates uncertainty from the initial assumptions and noisy measurements, thus providing the best estimate of the state of a system [1].

If the signal and noise are jointly Gaussian, then the Kalman Filter is an optimal Minimum Mean Squared Error (MMSE) estimator, as it minimizes the Mean Square Error (MSE) of the estimated state parameters [2]. Otherwise, it is an optimal Linear Minimum Mean Squared Error (LMMSE) estimator. If the noise is white, uncorrelated and has zero mean and the Kalman Filter is unbiased and consistent, then the Kalman Filter is the Best Linear Unbiased Estimator (BLUE). This is because it produces unbiased estimates of the state of a system with the minimum possible variance [3].

The Kalman Filter is based on two models: the Motion model and the Sensor model. The motion model describes how the state of the system changes over time, while the sensor model describes how the measurements of the system are related to the state [4].

Motion Model

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{w}_k \quad (1)$$

where $\mathbf{x}_k$ is the state estimate of the system at time step k , $\mathbf{x}_{k-1}$ is the initial state estimate of the system at time step $k-1$, $\mathbf{F}_k$ is the state transition matrix and $\mathbf{w}_k$ is the process noise that is assumed to be zero - mean Gaussian with the covariance $\mathbf{Q}_k$, i.e., $\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_k)$.

Sensor Model

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k \quad (2)$$

where $\mathbf{z}_k$ is the measurement of the system at time step k , $\mathbf{H}_k$ is the measurement matrix and $\mathbf{v}_k$ is the measurement noise that is assumed to be zero - mean Gaussian with the covariance $\mathbf{R}_k$, i.e., $\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k)$.

The Kalman Filter algorithm consists of two steps: the Time update or Prediction step and the Measurement update or Correction step. In the prediction step, the state estimate and error covariance of the current time step is predicted based on the updated state estimate and error covariance of the previous time step as defined in equations (3) and (4). In the correction step, the Kalman gain is computed first. Then, the updated state estimate and error covariance are calculated by using the current measurement to correct the predicted state estimate and error covariance and give a more accurate state estimate and error covariance for the current time step as defined in equations (5) (6) and (7) [5].

Prediction Step

$$\mathbf{x}_{pk} = \mathbf{F}_k \mathbf{x}_{k-1} \quad (3)$$

$$\mathbf{P}_{pk} = \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_k^T + \mathbf{Q}_k \quad (4)$$

where $\mathbf{P}_{k-1}$ is the initial covariance matrix of the state at time step $k-1$, $\mathbf{x}_{pk}$ is the predicted state estimate of the system at time step k and $\mathbf{P}_{pk}$ is the predicted error covariance of the state at time step k .

Correction Step

$$\mathbf{K}_k = \mathbf{P}_k \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{pk} \mathbf{H}_k^T + \mathbf{R}_k)^{-1} \quad (5)$$

$$\mathbf{x}_{uk} = \mathbf{x}_{pk} + \mathbf{K}_k (\mathbf{z}_k - \mathbf{H}_k \mathbf{x}_{pk}) \quad (6)$$

$$\mathbf{P}_{uk} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_p \quad (7)$$

where K_k is the gain matrix, x_{uk} is the updated state estimate of the system at time step k and P_{uk} is the updated error covariance of the state at time step k . The aforementioned steps of the Kalman Filter algorithm are shown in Figure. 1 [6]

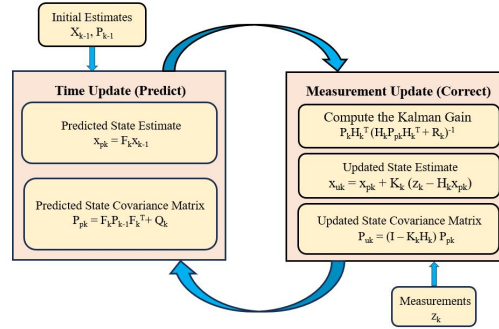


Figure 1. Kalman Filter algorithm

The Kalman Filter offers several advantages, including real time estimation, adaptability to non-linear systems, robustness to noise and efficient computation. However, it also has some disadvantages, such as sensitivity to initial conditions, assumptions of linearity, difficulty in turning parameters and computational complexity [8]. It is widely used in a variety of applications, including navigation, tracking, robotics, control systems and signal processing.

III. RESULTS

Fig. 2 illustrates the simulation of helical path using Kalman Filter, the red line represents the actual path and green line represents the predicted path. As seen in Fig. 2, the actual path and estimated path are very close to each other, thus providing better tracking accuracy. As the value of measurement noise increases, the actual path and estimated path diverge greatly, resulting in poor tracking accuracy. This comparison shows the accuracy of tracking performance of radar systems.

The Fig. 3 shows the plot of the relative predicted error values in X direction at various SNRs. Here the range of SNR varies from -20 to 60 dB. It is evident that when SNR increases, the relative predicted error in X direction drastically drops to zero. This is because the signal power rises as the noise power falls. As a result, the Kalman Filter works well for creating an ideal tracking system.

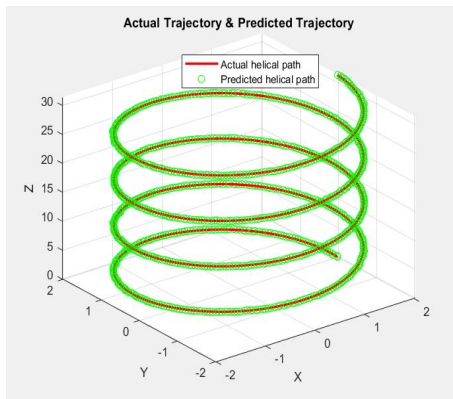


Figure 2. Simulation Result of Kalman Filter

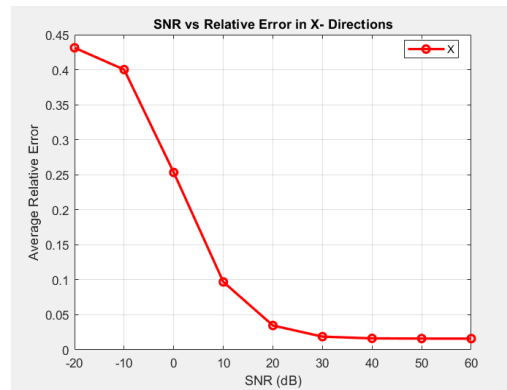


Figure 3. SNR Vs Relative Error in X direction

The Fig. 4 shows the plot of the relative predicted error values in Y direction at various SNRs. Here the range of SNR varies from -20 to 60 dB. It is evident that when SNR increases, the relative predicted error in Y direction drastically drops to zero. This is because the signal power rises as the noise power falls. As a result, the Kalman Filter works well for creating an ideal tracking system.

The Fig. 5 shows the plot of the relative predicted error values in Z direction at various SNRs. Here the range of SNR varies from -20 to 60 dB. It is evident that when SNR increases, the relative predicted error in Z direction

drastically drops to zero. This is because the signal power rises as the noise power falls. As a result, the Kalman Filter works well for creating an ideal tracking system.

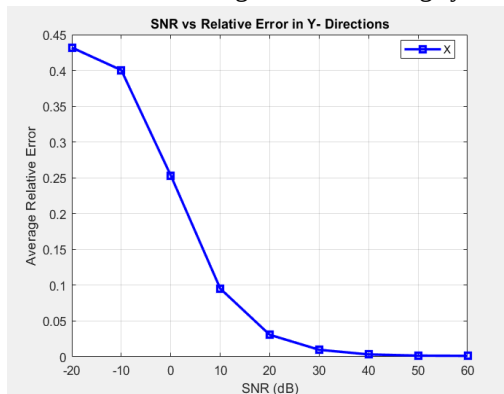


Figure 4. SNR Vs Relative Error in Y direction

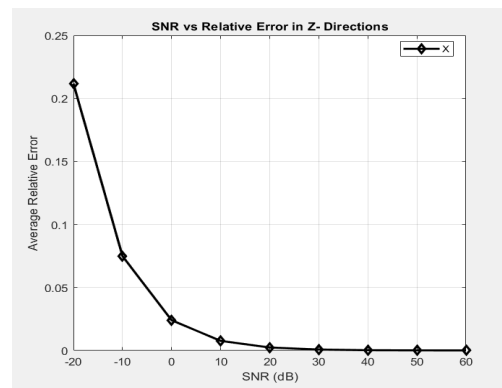


Figure 5. SNR Vs Relative Error in Z direction

IV. CONCLUSION

This paper focused on linear Kalman Filter for Radar Target Tracking. Recursive computations were used by the Kalman Filter to attain a solution of discrete linear filter. Being an adaptive filter, Kalman filter used a feedback loop to analyse the relation between its estimated value and measured value and attained the result after minimising the noises in the measured value. This research examined how measurement noise affects tracking performance of Radars in three dimensions in terms of relative prediction error at various SNR. The SNR vs relative predicted error was compared and concluded that the relative predicted error in all the three dimensions for the complex path we studied was very low and thus the proposed method provides a better tracking performance.

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