

A Comparative study of Morlet Wavelet Coherence & Complex Gaussian Wavelet Coherence between ECG and EEG Signals

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Abstract— Wavelet coherence is a time - frequency method which is used to analyze the degree of correlation between two signals. In this study the degree of correlation between ECG and EEG signals Using Wavelet coherence is calculated and a comparison using Morlet wavelet and Complex Gaussian wavelet is made. In this work, the results show that the Wavelet Coherence using Complex Gaussian wavelet gives significant and best value of coherence at all samples in the scale interval of 53,512 as compared to Wavelet Coherence using Morlet wavelet which shows that the analyzed signals are coherent at all samples but only in the scale interval of 53,365 which establishes that Wavelet- coherence technique using Complex Gaussian wavelet overcomes the limitations of conventional Coherence analysis by providing information both in temporal as well as in spectral domain.

Index Terms— ECG, EEG, Coherence, Wavelet Coherence, Morlet wavelet, Complex Gaussian wavelet.

I. INTRODUCTION

From the past few decades, lot of work has been done for the development of various signal processing methods which are aimed to measure and analyze the degree of coherence between biomedical signals [1], as they carry important information about the behavior of the living systems under study. A proper processing of these signals in principle enhances their physiological and clinical information. It is certainly true that basically all methods of signal processing have been applied to biomedical signals in various and differentiated experimental conditions. Many of these methods have been conceived in different research areas such as information processing, statistics, system and control theory, communication theory, medical informatics, lasers and electro-optics, digital signal, image analysis, etc. The degree of coherence between biomedical signals mainly highlights the association between them. It is very important in describing a biological system because of the presence of several mechanism co-operating and competing with each other within the system for its regulation. It shows the influence and relation of one signal over the other. It is commonly characterized by the time-series methods, namely Correlation (such as cross correlation), Phase synchrony (such as mean phase coherence) and Coherence (such as magnitude squared coherence (MSC) [2].

Coherence is a conventional method to analyze and find the linear relation between two signals by determining the correlation between their spectra [3]. Coherence is well known as “the frequency domain analog of the autocorrelation function”. However, the conventional coherence method which is based entirely on spectral

methods does not provide any information regarding temporal structure of the signal beyond phase information and so therefore it cannot give any information on dynamically varying signals. So, it assumes signals to be stationary [3]. Therefore, to overcome the shortcomings of conventional coherence (Fourier based), which are mainly caused by essentially neglecting time resolution for perfect frequency resolution, various new time–frequency signal analysis methods has been established as an important technique. Time–frequency signal analysis methods offer simultaneous Interpretation of the signal both in time and frequency which allows local, transient or intermittent components to be elucidated.

A number of time–frequency methods are currently available for the high-resolution decomposition in the time–frequency plane useful for signal analysis, including the short time Fourier transform (STFT), Wigner–Ville transform (WVT), Choi–Williams distribution (CWD) and the continuous wavelet transform (CWT). Of these the continuous wavelet transform has emerged as the most favored tool by researchers as it does not contain the cross terms inherent in the WVT and CWD methods while possessing frequency- dependent windowing which allows for arbitrarily high resolution of the high frequency signal components (unlike the STFT).

However, wavelet transform analysis began in the mid-1980s where it was developed to interrogate seismic signals. The application of wavelet transform analysis in science and engineering really began to take off at the beginning of the 1990s, with a rapid growth in the numbers of researchers turning their attention to wavelet analysis during that decade. The last few years have each seen the publication of over 1000 refereed journal papers concerning application of the wavelet transform. Wavelet based coherence for example, have already been used in other scientific contexts such as plasma physics and aerodynamics. In this work, focus is to measure the synchronization or association between the two most widely used biomedical signal-Electrocardiogram (ECG) and Electroencephalogram (EEG) using Wavelet based Coherence. The ECG measures the electrical activity of the heart due to contractile activity. The ECG measures the electrical activity of the heart due to contractile activity of the heart and EEG measures the electrical activity of the brain. This electrical activity is account of firing of millions of neurons within the brain and the resultant electrical signals are picked from multiple electrodes placed on the scalp.

For example, the study of neuron-synchronization of EEG signals can help us to understand the underlying cognitive processes. When task of learning is going on, cognitive processes take place in different brain regions. Investigating neural synchronization can help us to understand the underlying cognitive processes. The cognitive and information processing take place in different brain regions. In order to investigate how these distributed brain regions are linked together and the information is exchanged, the coherent analysis is frequently used as a tool for studying the relationship between two channel EEG [3]

II. COHERENCE

Coherence is the well–established standard and traditional approach for the analysis of biomedical signals. Coherence is defined as the absolute square of the cross-spectrum of two signals normalized by the product of their auto-spectra [5]. It includes the information of a cross-correlation function of phase angles between two signals at different frequencies. When the phase difference between two signals is constant then coherence is equal to 1 and when the phase difference between signals is random then coherence is equal to 0. Therefore, it lies in the range of 0 and 1. Coherence is mathematically analogous to correlation, however, coherence measures phase differences and yields a much finer measure between signals.

The one way of calculating coherence between two signals $x(t)$ and $y(t)$ is as follows:

1. First, to calculate the Fourier transform of each of the signals

$$X_i(f) = \frac{1}{2\pi} \int x(t) e^{-j2\pi ft} dt \quad (1)$$

$$Y_i(f) = \frac{1}{2\pi} \int y(t) e^{-j2\pi ft} dt \quad (2)$$

2. Next, the auto-spectrum of each signal and their cross-spectrum over each segment needs to be calculated.

$$P_{xx}(f, i) = X_i(f) X_i^*(f) \quad (3)$$

$$P_{xy}(f, i) = X_i(f) Y_i^*(f) \quad (4)$$

$$P_{yy}(f, i) = Y_i(f) Y_i^*(f) \quad (5)$$

In (3), (4) and (5) P_{xx} and P_{yy} represent the auto-spectra while P_{xy} represents the cross spectrum, with the $*$ operator indicates complex conjugation

3. Finally, the coherence between the two signals is given as

$$K(f) = \frac{\left| \sum_i P_{xy}(f, i) \right|^2}{\sum_i P_{xx}(f, i) \sum_i P_{yy}(f, i)} \quad (6)$$

As coherence is Fourier based, it suffers from some limitations which include that it does not give any information on dynamically varying signals, as it assumes signals to be stationary and it is very sensitive to fluctuations of linearity in phase.

In order to overcome the limitations of Fourier based coherence, Wavelet Coherence is the novel approach.

III. WAVELET COHERENCE

Wavelet Coherence is a modern time-frequency method, which incorporates Continuous wavelet transform and time-varying coherent analysis offering more flexibility.

Wavelet Coherence combines the concept of wavelet transform with the coherence analysis which employs an alternative way for quantifying synchronization of the signals with both temporal and spectral resolution. The wavelet coherent analysis can explore the amount of synchrony among multiple-channel signals, and are used to investigate the synchronization and the corresponding information processing of the various signals.

In this work, Wavelet coherence is implemented on biomedical signals. For the implementation of Wavelet coherence, ECG and EEG signals which are the most important and frequently used biomedical signals are used and are taken from online database of MIT-BIH.

Wavelet Coherence is a time-frequency method, which provides dynamic coherent analysis incorporating Continuous wavelet transform (CWT) overcoming the limitations of conventional coherence providing both temporal and spectral information. It provides more flexibility [1].

Wavelet Coherence is defined as

$$C(t, f) = \frac{\left| \sum_{i=-\Delta}^{\Delta} PW_{xy}(T, f, i) \right|^2}{\sum_i PW_{xx}(T, f, i) \sum_i PW_{yy}(T, f, i)} \quad (7)$$

Where T is the time around which the coherence

is calculated, i is the current index, and f is the frequency. The summations are carried around a variable segment size Δ , which is inversely proportional to frequency.

Also, the wavelet cross spectrum can be defined as:

$$PW_{xy}(t, f) = W_x(t, f) W_y^*(t, f) \quad (8)$$

As for higher frequencies wavelet transform uses a shorter window and for lower frequencies a longer window is used, also the coherence segment size Δ is also inversely proportional to frequency which makes this approach more suited to analyze and measure dynamic coherence. This is accomplished via a frequency-adaptive window in the time frequency plane [2].

Wavelet coherent spectrum is defined and computed from the cross-wavelet magnitude spectra which serves to indicate the degree of coherence and the cross-wavelet phase is used to provide the direction of Information flow between two signals [1][3].

Wavelet coherence is used to analyze the linear relation between two signals in both time and frequency. It provides both the temporal as well as spectral information. Hence, the signals do not have to be stationary [4]

IV. SIGNAL MEASUREMENT

The ECG and EEG signals are obtained from the MIT-BIH database [5] and the software used is MATLAB R2023a. The records taken are 100.dat and chb01_01.edf as shown in fig.1 and fig 2. The record of ECG 100.dat and EEG chb01_01.edf is having sampling frequency of 360 Hz and of 1 Hz respectively with 11-bit resolution. The duration of the signal is 10s and has 3600 samples but here for experiment only 1000 samples has been considered.

V. RESULTS

A. Implementation of Coherence between ECG & EEG Signals

Fourier based coherence is implemented on ECG and EEG Signals as shown in Fig.3. Maximum value of coherence obtained between ECG and EEG Signals is 0.6 i.e. the signals are 60% coherent near the Normalized frequency of 0.2 rad/sample.

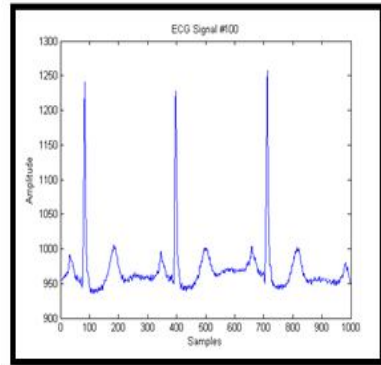


Fig.1. Electrocardiogram (ECG) Signal

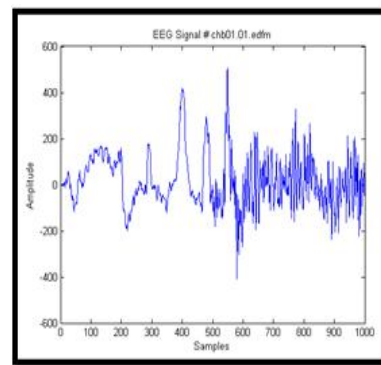


Fig.2. Electroencephalogram (EEG) Signal

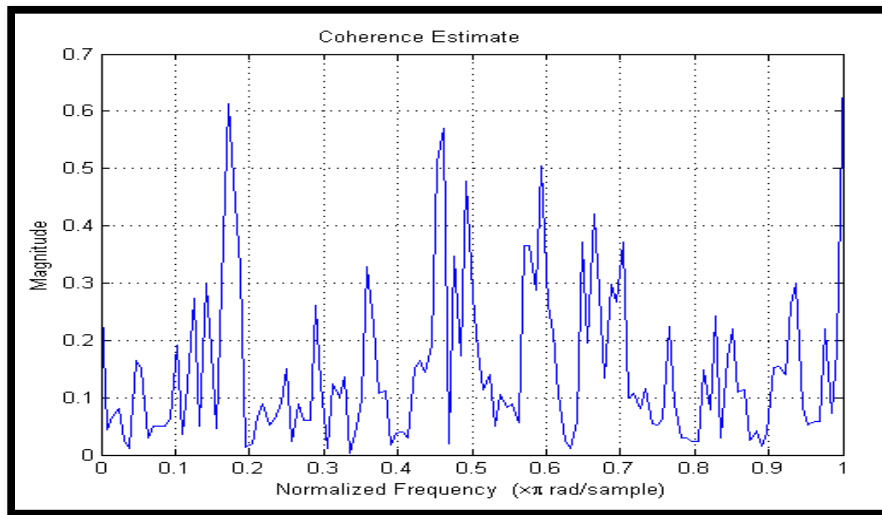


Fig.3. Coherence between ECG and EEG Signals

a. Establishing the Optimized wavelet

Wavelet coherence has many dependencies as it consists both the concepts of Wavelets and Coherence, therefore, the parameters affecting wavelets affects the overall Wavelet Coherence. Wavelet coherence depends on various parameters which are as:

- Wavelet family
- Scale
- Number of Wavelet coefficients.
- Number of Data points.
- Sampling Frequency.
- Noise.
- Frequency Bandwidth characteristics of Wavelets.

Based on the wavelet family, Complex Gaussian wavelet is established as optimized wavelet as it provides detailed information in transient signal detection than real-valued wavelets.

B) Implementation of Wavelet coherence between ECG & EEG Signals using Morlet Wavelet

The Morlet wavelet, named after Jean Morlet in 1984, is complex-valued, and consists of a Fourier wave modulated by a Gaussian envelope. Morlet wavelet is defined as:

$$\psi(\eta) = \pi^{-1/4} e^{j\omega t} e^{-\tau^2/2} \quad (9)$$

Where w is the central frequency of the mother wavelet.

In order to calculate Wavelet coherence, continuous wavelet transform and wavelet cross spectrum of the analyzed signals are also need to be calculated.

i. Continuous wavelet transform of ECG and EEG Signal

The Continuous wavelet transform of ECG and EEG Signals using Morlet wavelet is shown in Fig.4. For implementation 1000 samples of both ECG and EEG signals are taken and the wavelet scale is taken as 1:512.

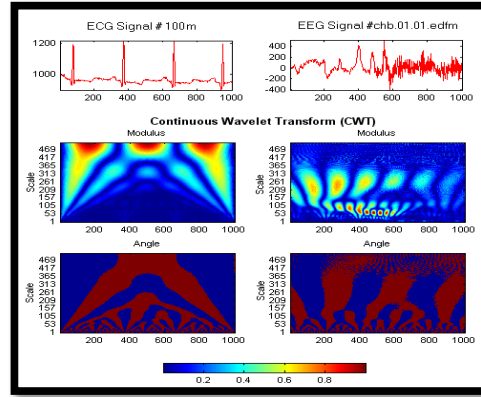


Fig.4.CWT of ECG and EEG Signals

ii. Wavelet cross spectrum of ECG and EEG Signals

The wavelet cross spectrum (WCS) between two time series $x(t)$ and $y(t)$ is defined as

$$C_{xy}(a,b) = X^*(a,b)Y(a,b) \quad (10)$$

where a and b are scale and time. $X(a,b)$ and $Y(a,b)$ are the continuous wavelet transform of the time series $x(t)$ and $y(t)$, respectively and $*$ means complex conjugate. The magnitude of the wavelet cross spectrum can be interpreted as the absolute value of the local covariance between the two time series in the time-scale plane. Fig.5. highlights the fact that both signals have a significant contribution around scale 261 at the sample value of 100.

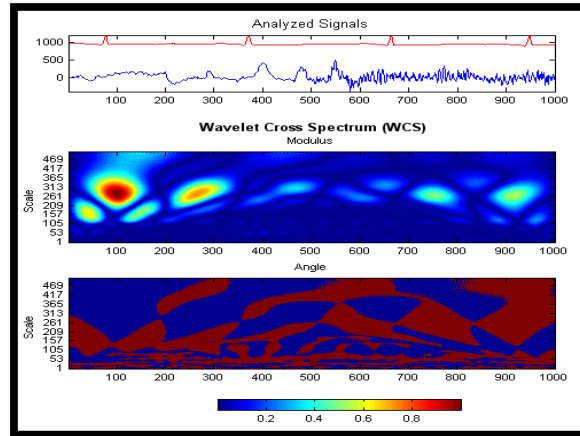


Fig.5 Wavelet Cross spectrum of ECG and EEG Signals

iii. Wavelet Coherence of ECG and EEG Signals using Morlet wavelet:

The wavelet coherence between the record 100.dat and chb01_01.edf using Morlet wavelet having scale of 1:512 is shown in Fig 6. The arrows in the figure represent the relative phase between the two signals as a function of

scale and position. As shown in Fig.6, the ECG and EEG signals are maximum coherent at all samples in the scale interval of [53,365].

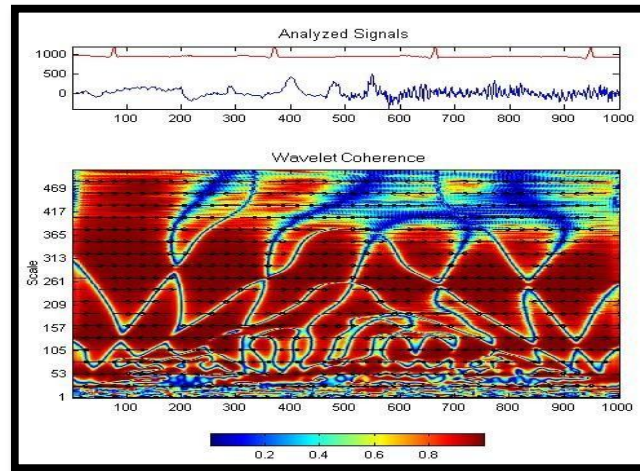


Fig.6.Wavelet Coherence using Morlet wavelet between ECG and EEG Signals

C) Wavelet Coherence using Complex Gaussian wavelet

Complex wavelets provide more detail information in transient signal detection than real-valued wavelets. Wavelet Coherence using Complex Gaussian wavelet is shown in Fig.7. As shown in Fig.7, the ECG and EEG signals are maximum coherent at all samples in the scale interval of [53,512].

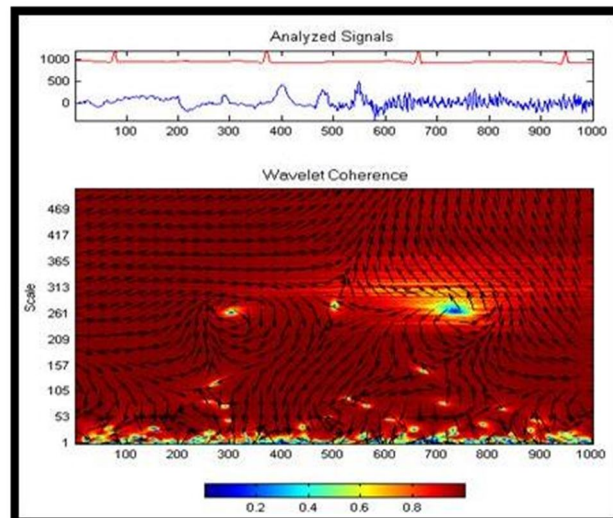


Fig.7. Wavelet Coherence using Complex Gaussian wavelet between ECG and EEG Signals

VI. DISCUSSION

The results show that the Wavelet Coherence using Complex Gaussian wavelet gives significant and best value of coherence at all samples in the scale interval of 53,512 as compared to Wavelet Coherence using Morlet wavelet which shows that the analyzed signals are coherent at all samples in the scale interval of 53,365 only. Also, in case of Complex Gaussian wavelet, there is also phase information, which can be used to infer the lead lag relationship between the signals. Therefore, as revealed by the results, Wavelet Coherence using Complex Gaussian wavelet gives best and optimized results as compared to Wavelet Coherence using Morlet wavelet. The performance is further enhanced by the use of complex wavelets as they provide more detailed information in transient signal detection than real-valued wavelets.

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