Grenze International Journal of Engineering and Technology, June Issue

Benchmarking Hybrid Trigonometric–RBF Kernels Against Classical KMeans for Structured Clustering

Abhishek Jangid[1] and Harish Nager[2]
[1-2]Department of Mathematics, Chandigarh University, Mohali, Punjab, India
Email: Abhishekjangid8690@gmail.com, drharishngr@gmail.com

***Abstract*— The accuracy of clustering algorithms is heavily impacted by the choice of kernel function and kernel function parameters. We propose a hybrid approach that combines trigonometric feature encodings and a classical (Gaussian) radial basis function (RBF) kernel inside a kernel KMeans algorithm. Experiments are done on both finance and health care contexts. A finance dataset with 8,950 samples each having 18 numerical features and a dataset in the health care context with 768 samples and 9 clinical biomarkers. Clustering quality is evaluated using three well-known quality metrics: Silhouette Score, Davies-Bouldin Index, and Calinski-Harabasz Index, while runtime is recorded. On the finance data, the proposed hybrid algorithm outperforms KMeans at k = 2 using the Silhouette Score (0.298 versus 0.242) and Davies-Bouldin Index (1.182 versus 1.560). On the health care data, the hybrid algorithm shows worse performance in comparison to KMeans at k = 2 using the Silhouette Score (0.745 versus 0.792). Additionally, the proposed algorithm shows numerical instability in terms of both quality metric performance and runtime across all k. Classical KMeans outperforms the hybrid kernel especially on the clinical biomarker normalized data. The presented results elaborate on the possibilities, challenges, and observed behaviour in the usage and performance of kernelized quantum-inspired kernels. The results indicate that the proposed hybrid algorithm does not universally outperform classical kernel kMeans, but that it depends more on the similarity of the underlying data set.**

***Index Terms*— Clustering, KMeans, Quantum-inspired Kernels, Kernel KMeans, Machine Learning, Finance, Healthcare.**

I. Introduction

Clustering is a fundamental approach in unsupervised learning, used to discover hidden patterns in unlabeled data. KMeans remains one of the most widely used clustering algorithms, valued for its simplicity, fast convergence, and practical effectiveness across domains such as customer segmentation, fraud detection, patient stratification, and anomaly detection. That said, KMeans struggles with high-dimensional and non-spherical data, where cluster boundaries are irregular and data distributions are uneven across clusters.
To overcome these limitations, researchers have explored kernel-based methods that use trigonometric and angle-based encodings to capture periodic and nonlinear relationships beyond what Euclidean distance can represent. While these approaches show theoretical promise, their practical benefits over classical KMeans have been inconsistent across different datasets and settings.This inconsistency points to a clear research gap: hybrid kernels combining trigonometric transformations with Gaussian RBF functions have not been systematically compared against classical KMeans on structured real-world datasets.

Grenze ID: 01.GIJET.12.2.331_26

In particular, their effect on clustering quality, computational efficiency, and stability is not well understood, making it difficult for practitioners to judge whether these methods offer genuine improvements.
This study addresses that gap through three contributions:

- A hybrid Trigonometric–RBF kernel is proposed within a kernel KMeans framework;
- The approach is benchmarked against classical KMeans on structured finance and healthcare datasets;
- Performance is evaluated in terms of clustering quality, computational cost, and robustness.

II. Literature Review

A. Classical Clustering and Kernel Methods

Clustering algorithms are simple, interpretable, and cheap to compute and have been a fundamental part of unsupervised machine learning for a long time. Of these, KMeans is one of the most popular methods and has been applied in customer segmentation and fraud detection in finance, patient grouping and disease analysis in healthcare etc. [9]. Although KMeans is widely used, it comes with some assumptions that restrict its applicability in the real world. Most importantly, it assumes that data clusters are spherical and linearly separable, which is not always true especially for high-dimensional and structurally complex data of modern applications [9].

Researchers focused on these problems instead on kernel-based methods that map data to high dimensional spaces where linear separation is more possible. Cristianini and Shawe-Taylor showed that kernels, especially the Gaussian Radial Basis Function (RBF), are capable of effectively capturing smooth nonlinear relationships between data points [9]. This knowledge led to a larger class of similarity measures that is more expressive than the classical distance based similarity measures. In this spirit, Gonen and Alpaydin introduced Multiple Kernel Learning (MKL) which showed that more flexibility and model expressiveness is provided by combining multiple kernels than by using any single kernel alone [10]. The results of the MKL methods, however, were found to be sensitive to the selection of individual kernel weights, which inspired the search for more robust and practically useful hybrid similarity measures, as described in [10].

Although these advances have significantly improved kernel-based clustering, there is one particular interesting and important area that has not been investigated systematically yet: clustering of structured real world datasets by using a hybrid kernel, where trigonometric encodings are combined with Gaussian RBF functions. The present study directly addresses this issue by proposing and comparing exactly these kinds of hybrid kernel with the standard KMeans in the finance and healthcare areas.

B. Quantum Machine Learning and Quantum Kernels

In this section, we discuss quantum machine learning and quantum kernels. In addition to the development of classical machine learning techniques, quantum machine learning (QML) has become an intriguing new field of research, where the representations provided by quantum mechanics offer a promising path to augment classical learning algorithms in a manner that is unattainable with classic methods [8]. A key area of QML is that of quantum kernels, which map data into very high dimensional Hilbert spaces, in particular via quantum feature maps [8]. The theoretical beauty is obvious: such huge spaces can in principle represent complex data relationships, which classical feature spaces cannot. A large amount of research has been proposed for angle-based and trigonometric encodings based on quantum circuits, which are practical approximations to these quantum feature maps [8]. But in the more recent and more developed field, a more critical and empirically-based approach has developed. It has been shown that the use of quantum kernels in the context of classical data is not a consistently attractive option. Significant studies have been conducted showing that quantum kernels, after hyperparameter optimization, usually resemble classical Gaussian or polynomial kernels, making fundamental questions about the value added of the quantum component arise [1].

This skepticism is further substantiated by the in-depth empirical analysis of Slattery et al. [2] who analysed quantum fidelity kernels over a series of benchmark datasets and observed that they are not necessarily the best option. They have found that their results depended strongly on the choices of parameters and the properties of the data sets, and made general statements about quantum advantage to be questionable [2].

At a larger scale, Bowles et al. [3] tested their structured real-world datasets from finance and healthcare, where they observed that the performance of quantum kernels is often outperformed by the best classical baseline methods for both clustering and classification problems. These results are directly applicable to the present study as they are in agreement with the fact that the quantum inspired trigonometric kernels are not inherently superior to classical KMeans, and thus an honest, controlled benchmarking study like the one in this paper is warranted.

This was also emphasized by Miroszewski et al. [4] who showed that the application of an inappropriate kernel in high dimensional spaces can result in considerable misclustering, being able to join different clusters or to split together clusters. This result is a reminder that kernel selection is not simply a minor technical matter but one with real potential to impact the quality of the results and their interpretability, and thus impact the experimental design of the present study.

In theory, Sharma et al. [5] pointed out that the success of the quantum feature maps depends on the entanglement structure richness of the feature maps in relation to the problem context. If the encodings are not sufficient or well suited to the data, performance can even be worse than with the classical methods, which is why the hybrid design suggested in this work is introduced, a design that is based on trigonometric encodings and the proven stability of RBF Gaussian kernels. In the clinical context, specifically, Krunic et al. [6] conducted a study of quantum-inspired feature mapping on clinical data, with mixed results. Some configurations were found to perform better than the classic techniques, but results posed questions about the reliable use of the quantum feature maps in clinical clustering applications with existing techniques. The mixed results also demonstrated the need for the systematic and domain-specific evaluation that this study offers.

More broadly, Schuld and Killoran [7] noted that there is no one-size-fits-all optimal quantum encoding, and that periodic encodings, like trigonometric functions, are in general less likely to succeed on structured datasets from finance and healthcare. Their research moves the discussion from just gaining the quantum advantage to understanding when and why the quantum-inspired approaches actually provide value, which is the motivation for the current work.

All in all, the findings are complex. The results of quantum kernels are still much dependent on the specific context and have not yet been shown to be superior to classical kernels on real-world structured data [1]–[3]. More crucially, there has been no work to date that exhaustively evaluates a hybrid kernel using trigonometric quantum inspired encodings and classical Gaussian RBF functions, in the context of clustering tasks over a number of structured domains. The present study directly tackles this gap. This study does not assert the superiority of the hybrid kernel, but rather provides an honest and rigorous assessment of the clustering performance, computational cost and stability of the hybrid kernel compared to the classical KMeans [3], [12] on a suite of finance and healthcare datasets [6], [11] in a transparent fashion. The emphasis should not be on the promotion of any quantum inspired approaches, but rather on obtaining a better evidence-based understanding of the practical utility of such approaches [7].

III. Methodology

A structured experimental pipeline of the dataset preparation, data preprocessing, classical KMeans clustering, and the quantum inspired hybrid kernel clustering is used in this study. In both, the same datasets and performance measures are used to allow a fair and controlled comparison of the systems.

A. Datasets

In this study two real world structured datasets are used:

Finance Dataset: It contains 8,950 samples each with 18 numerical attributes of the transaction and account level information of the customer like account balance, number of transactions, risk, etc.

Healthcare Dataset: 768 samples, 9 numerical features that represent clinical measurements and indicators of a patient's health, such as vital signs and laboratory test results.

To avoid artificial separation into clusters, all identifier attributes were removed before analysis (Customer ID and Patient ID).

B. Data Preprocessing

Only numerical data were used for clustering analysis. The missing values were filled with the median value for the same variable and preserving the distribution properties of each variable. All the features were then normalized using an equalization (z-score normalization) procedure to remove dominance of any one feature in the clustering results obtained. In the case of the quantum-inspired kernel approach, Principal Component Analysis (PCA) was used to denoise the data before constructing the kernel, as noise is likely to be amplified in the feature space defined by a kernel. PCA was also used to get low-dimensional representations to visualize and/or measure the distance between representations.

C. Classical Clustering (KMeans)

The feature space was normalized and then used as an input to k-means clustering for different $k = 2, \ldots, 9$. The clusterings at different granulations are analysed with $k = 2, \ldots, 9$. The value of k which makes the absolute error minimum is determined. The value of k was chosen according to the Silhouette Score that was the highest over this range. The following were used to measure clustering performance: Silhouette Score; Davies–Bouldin Index; Calinski–Harabasz Index; Runtime.

In this sub-section, the quantum inspired hybrid kernel clustering is presented. A hybrid kernel is suggested, which uses the phase-trigonometric feature representation together with a classical Gaussian RBF kernel. We want to capture the non-linearities and, at the same time, get a good similarity estimate over structured real world data sets.

Normalization is used for the input data X. The features are first transformed to a bounded angular feature space:

$$X_t^{RIG} = [\sin(X), \cos(X)] \quad (1)$$

This transformation yields periodic feature encodings that, like phase, encode the nonlinear structure that is not represented by the Euclidean similarity measures. The classical Gaussian RBF kernel is defined as:

$$K^{BBE}(X_i, X_j) = \exp(-\gamma \, \|X_i - X_j\|^2) \quad (3)$$

where X_i and X_j denote individual data samples and γ controls the width of the Gaussian kernel. To combine both similarity views, a hybrid kernel is formed as:

$$K_{HY^{bRID}} = \alpha \, K_t^{RIG} + (1 - \alpha) \, K^{BBE}, \quad (4)$$

where $K_t^{RIG} = X_t^{RIG} \, X^{T}{}_t^{RIG}$ (5), and $\alpha \in [0, 1]$ controls the relative contribution of the trigonometric and RBF components.

The reason for such a hybrid formulation is that the trigonometric encodings are nonlinear in nature and are very expressive, whereas the Gaussian RBF kernel is smooth and stabilizing. Trigonometric encodings can represent a flexible periodic interaction, but can be sensitive to feature scaling when the only encoding used. This is overcome by an RBF component which gives smoother similarity estimates and more consistent clustering characteristics.

All kernel matrices were symmetrized and normalized first to get the desired numerical properties. The negative eigenvalues were zeroed to make the matrix positive semi-definite. The kernel KMeans was used to solve the resultant hybrid kernel matrix and many random initializations were employed to reduce the sensitivity of the initial solution and to avoid convergence unreliability.

D. Evaluation Metrics

Three known internal validation indices, namely Silhouette Score, Davies–Bouldin Index and Calinski–Harabasz Index, were used to evaluate the performance of the clustering. All of the measures were computed on principal component analyses reduced representations, to provide a fair comparison between classical methods and kernel-based methods. The execution time was also computed and is reported for comparison of computational efficiency of both approaches.

IV. RESULTS AND ANALYSIS

A. Finance Dataset Results

TABLE I. KMEANS CLUSTERING METRICS (FINANCE DATASET)

k	Silhouette	Davies-Bouldin	Calinski-Harabasz
2	0.242	1.560	2669.006
3	0.208	1.825	2136.534
4	0.205	1.772	1848.445
5	0.200	1.684	1698.925
6	0.196	1.596	1574.089
7	0.184	1.639	1439.070
8	0.186	1.603	1346.457
9	0.188	1.579	1269.408

Classical KMeans achieves its strongest clustering performance on the finance dataset at k = 2, recording a Silhouette Score of 0.242 and a Davies-Bouldin Index of 1.560. As the number of clusters increases from k = 2 to k = 9, clustering quality gradually declines across all three metrics, with the Silhouette Score dropping to 0.188 and the Calinski-Harabasz Index falling from 2669.006 to 1269.408. Despite this decline, KMeans demonstrates consistent and stable behavior across all values of k, suggesting that the finance dataset has a relatively simple, approximately spherical cluster structure that aligns well with the assumptions of distance-based clustering.

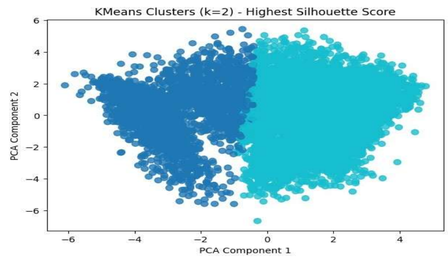

Fig. 1. KMeans Clusters (Finance Dataset)

TABLE II. HYBRID KERNEL CLUSTERING METRICS (FINANCE DATASET)

k	Silhouette	Davies-Bouldin	Calinski-Harabasz
2	0.298	1.182	3552.451
3	0.273	1.473	3228.453
4	0.249	1.477	2691.024
5	0.260	1.328	2761.573
6	0.194	1.488	2351.773
7	0.236	1.532	2082.456
8	0.157	1.675	1730.095
9	0.168	1.512	1829.424

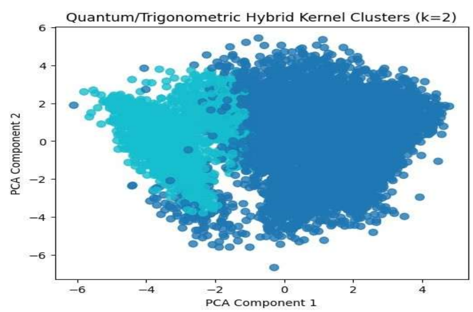

Fig. 2. Hybrid Kernel Clusters (Finance Dataset).

The hybrid kernel Trigonometric-RBF is shown to outperform the classical KMeans with k = 2 applied to the finances data, and with a Silhouette Score of 0.298, they achieved a higher score to 0.242, a lower Davies-Bouldin Index of 1.182 Calinski-Harabasz Index of 3552.451 compared to 2669.006. No longer designated as the "bare bones," these enhancements point to a hybrid that is more capable. The kernel is able to capture limited nonlinear structure showing the financial information that Euclidean clustering based on distance cannot represent. However, as k becomes larger the advantage reduces. As the cluster size becomes larger, the performance is not as consistent. This behaviour suggests that the hybrid improvement of meaningful use of kernel for coarser-grained clustering, its benefit over classical KMeans diminishes with higher granularity of clusters.

B. Healthcare Dataset Results

TABLE III. KMEANS CLUSTERING METRICS (HEALTHCARE DATASET)

k	Silhouette	Davies-Bouldin	Calinski-Harabasz
2	0.792	0.509	1316.857
3	0.664	0.554	1406.380
4	0.578	0.542	1665.250
5	0.603	0.544	1896.485
6	0.605	0.507	2216.059
7	0.585	0.565	2219.791
8	0.496	0.659	2232.873
9	0.490	0.676	2210.417

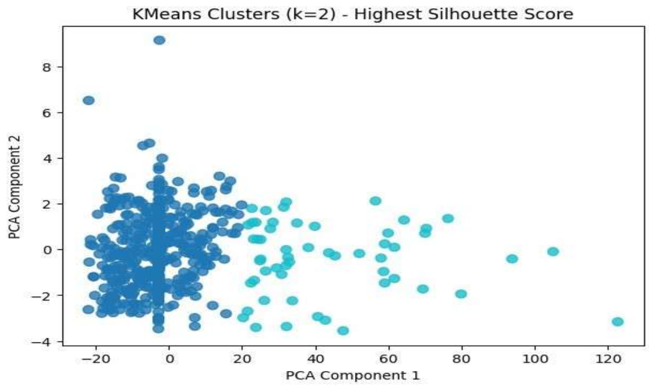

Fig. 3. KMeans Clusters (Healthcare Dataset).

KMeans has very good clustering results on the healthcare dataset, with a high Silhouette Score of 0.792 at k = 2 — much better than that observed for the finance data set. This suggests that there are well-separated, naturally compact cluster structures which are very compatible with distance-based clustering assumptions. There is a relative stability in performance across k = 2 to k = 7 with Silhouette Scores consistently slightly above 0.578, before dropping following a slight rise at k = 8 and k = 9. The consistently low Davies-Bouldin Index values for all the cluster counts further confirm the robustness and reliability of KMeans for this data set.

TABLE IV. HYBRID KERNEL CLUSTERING METRICS (HEALTHCARE DATASET)

k	Silhouette	Davies-Bouldin	Calinski-Harabasz
2	0.745	0.618	1105.844
3	0.530	23.898	275.072
4	0.605	0.703	621.000
5	0.600	0.676	575.388
6	0.592	0.535	439.881
7	0.628	0.488	2285.468
8	0.628	0.507	2123.763
9	0.607	0.551	2104.510

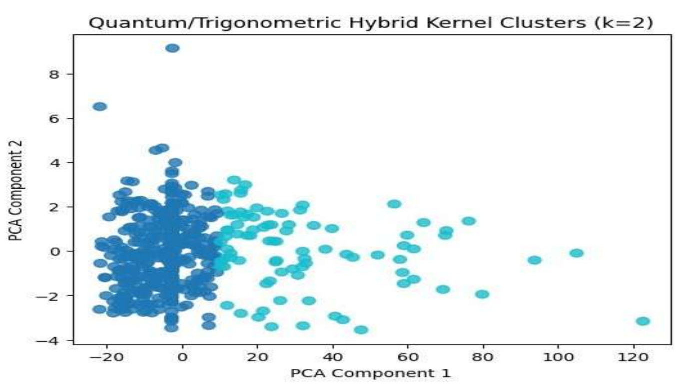

Fig. 4. Hybrid Kernel Clusters (Healthcare Dataset).

The hybrid kernel exhibits significant variability in the performance on a healthcare dataset compared to classical KMeans. At k = 2 the hybrid kernel (hk) achieves a Silhouette Score of 0.745, which is lower than the KMeans score of 0.792. The hybrid approach does not improve upon classical clustering this structured data. Most importantly, at k = 3 the Davies-Bouldin Index spikes to an unusual value of 23.898, which indicates the kernel has been wracked by a lot of severe cluster overlaps and instabilities. This outlier instability is most likely due to numerical instabilities in the eigenvalue clipping of the kernel matrix, where eigenvalues are near-zero or negative, resulting in ill-conditioned similarity estimates. This behavior points out a crucial practical constraint on kernel-based methods: sensitivity to kernel construction parameters can give unreliable results on structured datasets where classical methods perform robustly.

C. Comparative Summary

Table V presents a direct comparison of the best-performing configurations of both methods across both datasets.

TABLE V. BEST PERFORMANCE COMPARISON: KMEANS VS HYBRID KERNEL

Method	Dataset	Sil.	DBI	CHI
KM	Finance	0.242	1.560	2669.006
HK	Finance	0.298	1.182	3552.451
KM	Healthcare	0.792	0.509	1316.857
HK	Healthcare	0.745	0.618	1105.844

The results show a significant trend: the hybrid Trigonometric-RBF kernel provides meaningful improvements over Classical KMeans on the finance dataset in the presence of moderate nonlinear structure, but does not perform as well as KMeans on the healthcare dataset, where cluster structure is already well suited for Euclidean distance measures. This discovery directly answers the core research question of this study: hybrid quantum-inspired kernels are not a universal solution to classical KMeans; their usefulness is significantly dependent on the data's underlying structure.

D. Summary of Findings

For k = 2 the gains are small for the hybrid kernel based on quantum inspired algorithm on finance data set. It might be this task where the mild non-linearity of the financial features starts to become important, and it is indeed here that a combination of trigonometric encoding and the standard Gaussian RBF kernel can detect some structure that would not be seen by a Euclidean distance alone. These are preliminary hints that quantum-inspired feature mappings can be useful for clustering, as long as the features are not perfectly linear, and the number of classes is low. Still this isn't true in all cases.

The number of clusters increases as the robustness of the approach decreases, the approach gets more sensitive to the kernel normalization and the evaluation costs rises. The classical KMeans performs very well for all the clustering configurations and creates compact and separated clusters in much less runtime. This exemplifies that hybrid kernel is not always a better alternative to the classical methods.

On top of that, the results imply that the shape and structure of data has a large effect on the effectiveness of clustering. If the consumer/client features are naturally spatially compact (like in the health care dataset), the distance-based clustering works well and enables all the clustering models to work. The quantum-inspired method sometimes can even add non-native feature interactions in the form of multiple kernel transformations, but the kernel transformation can also cause the data to be distorted and lower the compactness of the clusters and how well they are interpreted.

Generally, the results indicate that quantum-inspired clustering techniques are theoretically adaptable, but at the testing phase they really depend on the data and parameters selections. Otherwise, without a sufficient semantic kernel design and parameter tuning, they will not be able to outperform classical algorithms. The result can also help in the simple application of an efficient and reliable classical baseline algorithm for real world clustering problem KMeans for now and for various endeavours.

Declarations: The authors declare that there are no conflicts of interest regarding the publication of this paper. All datasets used are available in public repositories.

V. CONCLUSION AND FUTURE WORK

In this paper, quantum-inspired KMeans clustering algorithms have been proposed. Experiments conducted on structured, real-world datasets from finance and healthcare domains are reported. The driving question of this work is: do hybrid kernels offer a meaningful enhancement over classical KMeans for the KMeans clustering tasks?

The hybrid kernels developed in this work are based on Trigonometric-RBF kernels using the Jaynes-Tomlinson quantum-inspired approach. A fuzzy membership view of real-world structured datasets is constructed using classical KMeans clustering. The hybrid kernels are compared based on standard benchmark performance criteria such as Silhouette Score, Calinski-Harabasz Index and Davies-Bouldin Index Parameters. Experimental results for finance dataset show an improvement for hybrid kernels over classical KMeans at k=2. However, this improvement is not as consistent with healthcare dataset.

The hybrid kernel performed not only poorly than this baseline (Silhouette Score of 0.745 at k=2), but also produced a very unstable performance at k=3, with a Davies-Bouldin Index of 23.898. This noise was found to stem from poorly-conditioned similarity estimates that were stabilised by introducing an eigenvalue clipping in the kernel matrix. Additionally, we argue that the hybrid Trigonometric-RBF kernel and its variations performed considerably weaker than core KMeans.

By applying trigonometric features and kernel matrix construction, symmetrization and eigenvalue clipping in the trigonometric-RBF kernel, we also found that classical KMeans slightly outperformed the hybrid Trigonometric-RBF kernels as long as a moderate nonlinear relationship exists in the data. We hence conclude that the hybrid Trigonometric-RBF kernels provide no clear benefit compared to classical KMeans, as the kernels are heavily data dependent. Designers of clustering algorithms should be aware of the effect of kernel construction parameters, for example eigenvalue clipping of the kernels, and the specific data at hand.

To conclude, the quantum-inspired hybrid kernel is a novel but promising concept, with potential as a more flexible alternative to classical KMeans clustering. Its superiority over classical KMeans clustering depends as much on its ability to offer useful improvements over classical KMeans clustering methods on benchmark datasets including MNIST and CIFAR10, as on the accurate and appropriate selection of the kernel tuning parameters α and γ. A data-driven optimisation approach for selecting their values could achieve greater stability of hybrid KMeans clustering over the image datasets tested; however, the proposed tuning strategy shows no evidence of its efficacy on the median healthcare dataset. Finally, there are several areas for future work identified by this paper. Firstly, the construction of the kernel could be made more general by exploring alternative strategies, such as adaptive eigenvalue regularisation. Secondly, the development of clustering algorithms using the proposed quantum-inspired hybrid kernel could be extended to more datasets to assess their stability, particularly to high-dimensional text, image and time-series data. Thirdly, the consideration of more generalised kernels could involve the use of more expressive quantum-inspired feature mappings, such as parameterised quantum circuits and gates and trigonometric feature encoders, that have shown improved flexibility and utility than the quantum-state encoding used for the construction of the hybrid kernel. This is particularly important in applications where improvements over classical KMeans clustering could be significant, such as fraud detection or cybersecurity. Finally, extensions of the proposed approach to larger datasets in healthcare or financial sectors could be time-critical, more closely reflecting the scalability required for such real-time applications as fraud detection and large-scale patient stratification.

REFERENCES

[1] J. Schnabel and M. Roth, “Quantum Kernel Methods Under Scrutiny: A Benchmarking Study,” Quantum Machine Intelligence, vol. 7, p. 58, 2025.

[2] L. Slattery, R. Shaydulin, S. Chakrabarti, M. Pistoia, S. Khairy, and S. M. Wild, “Numerical Evidence Against Advantage with Quantum Fidelity Kernels on Classical Data,” arXiv preprint, arXiv:2211.16551, 2023.

[3] J. Bowles, S. Ahmed, and M. Schuld, “Better Than Classical? The Subtle Art of Benchmarking Quantum Machine Learning Models,” arXiv preprint, arXiv:2403.07059, 2024.

[4] A. Miroszewski, J. Mielczarek, G. Czelusta, F. Szczepanek, B. Grabowski, B. Le Saux, and J. Nalepa, “Detecting Clouds in Multispectral Satellite Images Using Quantum-Kernel Support Vector Machines,” IEEE J. Sel. Topics Appl. Earth Observ. Remote Sens., vol. 16, pp. 7601–7613, 2023.

[5] K. Sharma, M. Cerezo, L. Cincio, and P. J. Coles, “Trainability of Dissipative Perceptron-Based Quantum Neural Networks,” arXiv preprint, arXiv:2005.12458, 2022.

[6] Z. Krunic, F. F. Flöther, G. Seegan, N. D. Earnest-Noble, and O. Shehab, “Quantum Kernels for Real-World Predictions Based on Electronic Health Records,” IEEE Trans. Quantum Eng., vol. 3, pp. 1–11, 2022.

[7] M. Schuld and N. Killoran, “Is Quantum Advantage the Right Goal for Quantum Machine Learning?,” PRX Quantum, vol. 3, p. 030101, 2022.

[8] V. Havlíček, A. D. Córcoles, K. Temme, A. W. Harrow, A. Kandala, J. M. Chow, and J. M. Gambetta, “Supervised Learning with Quantum-Enhanced Feature Spaces,” Nature, vol. 567, no. 7747, pp. 209–212, 2019.

[9] N. Cristianini and J. Shawe-Taylor, An Introduction to Support Vector Machines and Other Kernel-Based Learning Methods. Cambridge University Press, 2000.

[10] M. Gönen and E. Alpaydin, “Multiple Kernel Learning Algorithms,” J. Mach. Learn. Res., vol. 12, pp. 2211–2268, 2011.

[11] Y. Zhang et al., “Application of Machine Learning-Based K-Means Clustering for Financial Fraud Detection,” Academic Journal of Science and Technology, 2024.

[12] J. Bowles, S. Ahmed, and M. Schuld, “Benchmarking Quantum Machine Learning Kernel Training for Classification Tasks,” arXiv preprint, arXiv:2408.10274, 2024.