Grenze International Journal of Engineering and Technology, June Issue

Randic Status Energy of Graph

Kiran Paul[1], K. N Prakasha[2], M. A Sriraj[3], S. Vijay[4], M. R Raghavendra[5], P. Siva Kota Reddy[6], P. Purushotham[7] and Arun C Dixit[8]
[1-3]Department of Mathematics, Vidyavardhaka College of Engineering, Affiliated to Visvesvaraya Technological University, Mysuru-570002, India
Email: prakashamaths@gmail.com
[4]Department of Mathematics, Government Science College, Hassan, India
[5]Department of Mathematics Dayananda Sagar Academy of Technology & Management Udayapura, Kanakapura Road, Bengaluru-560082
[6]Department of Mathematics, JSS Science and Technology University, Mysuru-570 006, India
[7]Department of Mathematics, Maharaja Institute of Technology Mysore, Belawadi, Mandya-571 477, India
[8]Department of Mechanical Engineering, Vidyavardhaka College of Engineering, Affiliated to Visvesvaraya Technological University, Mysuru-570002, India

***Abstract*—In this paper, we introduce and investigate the Randic status energy of graph which is based on the status of the vertex. We obtain the upper and lower bounds also. We discussed the newly introduced status-based energy for well-known and much studied standard graph structures.**

***Index Terms*— Randic status index, Randic status energy.**

I. Introduction

In recent years, many topological indices which are based on the degrees of the vertices are introduced. Randic status index has been introduced by K. N. Prakasha [4] and the index depends on the status of the vertex. For new topological indices, we suggest the reader to refer the papers [9-20]. In Graph theory, status of the vertex 'u' is defined as the sum of distances of all other vertices from 'u'.
Randic status index is defined as

$$RS(G) = \frac{1}{\sqrt{\sigma(u) \cdot \sigma(v)}}$$

where σ_u represents the status of the vertex u.
Motivated by the Randic Status index, we introduce the $Randic -$ Status matrix $RS(G)$ as $RS(G) = (RS)_{n\times n}$

$$RS(G) = \begin{cases} \dfrac{1}{\sqrt{\sigma(u) \cdot \sigma(v)}} & if\ v_i\ connected\ with\ v_j \\ 0 & otherwise \end{cases}$$

Matrix representation of a graph set a new trend in the research of Mathematical Chemistry. Adjacency matrix is also known as 0-1 matrix whose entries will be 1, if the two corresponding vertices are adjacent and for other case the entry will be 0. Energy of graph is the sum of absolute values of eigenvalues extracted by the adjacency

Grenze ID: 01.GIJET.10.2.478

matrix of graph. Thousands of studies have been published since the beginning of graph energy (See for example [1-3, 5-8]).
The Randic status energy is given by

$$RSE(G) = \sum_{i=1}^{n} |\alpha_i|$$

II. RANDIC STATUS ENERGY OF GRAPH STRUCTURES

Theorem 1. *Randic status energy of complete graph K_n is $RSE(K_n) = 2$.*

Proof. For each and every vertex u in K_n, $\sigma_{(u)} = (n-1)$. Then the every ij^{th}-entry of the Randic status matrix will be $\frac{1}{n-1}$.

$$RS(K_n) = \begin{bmatrix} 0 & \frac{1}{n-1}_{n-1\times n-1 times} \\ \frac{1}{n-1}_{n-1\times n-1 times} & 0 \end{bmatrix}.$$

$$\begin{vmatrix} \alpha & -\frac{1}{n-1}_{n-1\times n-1 times} \\ -\frac{1}{n-1}_{n-1\times n-1 times} & \alpha \end{vmatrix} = 0.$$

Here the spectrum will be having the eigen values $-\frac{1}{n-1}$ ($n-1$ times) and 1 (one time).
Therefore, $RSE(K_n) = 2$. □

Theorem 2. *The Randic status energy of Crown graph S_n^0 is $RSE(S_n^0) = \frac{4(n-1)}{3n}$.*

Proof. In Crown graph each vertex will be having the status $3n$. The Randic status matrix is

$$\begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & \frac{1}{3n} & \dots & \frac{1}{3n} & \frac{1}{3n} \\ 0 & 0 & 0 & \dots & 0 & \frac{1}{3n} & 0 & \dots & \frac{1}{3n} & \frac{1}{3n} \\ 0 & 0 & 0 & \dots & 0 & \frac{1}{3n} & \frac{1}{3n} & \dots & 0 & \frac{1}{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & \frac{1}{3n} & \frac{1}{3n} & \dots & \frac{1}{3n} & 0 \\ 0 & \frac{1}{3n} & \frac{1}{3n} & \dots & \frac{1}{3n} & 0 & 0 & \dots & 0 & 0 \\ \frac{1}{3n} & 0 & \frac{1}{3n} & \dots & \frac{1}{3n} & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{3n} & \frac{1}{3n} & 0 & \dots & \frac{1}{3n} & 0 & 0 & \dots & 0 & 0 \\ \frac{1}{3n} & \frac{1}{3n} & \frac{1}{3n} & \dots & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}.$$

$$|\alpha I - RS_3(K_n)| =$$

$$\begin{vmatrix} \alpha & 0 & 0 & \cdots & 0 & 0 & -\frac{1}{3n} & \cdots & -\frac{1}{3n} & -\frac{1}{3n} \\ 0 & \alpha & 0 & \cdots & 0 & -\frac{1}{3n} & 0 & \cdots & -\frac{1}{3n} & -\frac{1}{3n} \\ 0 & 0 & \alpha & \cdots & 0 & -\frac{1}{3n} & -\frac{1}{3n} & \cdots & 0 & -\frac{1}{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -\frac{1}{3n} & -\frac{1}{3n} & \cdots & -\frac{1}{3n} & 0 \\ 0 & -\frac{1}{3n} & -\frac{1}{3n} & \cdots & -\frac{1}{3n} & \alpha & 0 & \cdots & 0 & 0 \\ -\frac{1}{3n} & 0 & -\frac{1}{3n} & \cdots & -\frac{1}{3n} & 0 & \alpha & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{1}{3n} & -\frac{1}{3n} & 0 & \cdots & -\frac{1}{3n} & 0 & 0 & \cdots & \alpha & 0 \\ -\frac{1}{3n} & -\frac{1}{3n} & -\frac{1}{3n} & \cdots & 0 & 0 & 0 & \cdots & 0 & \alpha \end{vmatrix} = 0.$$

Characteristic equation is

$$\left(\alpha - \frac{1}{3n}\right)^{n-1}\left(\alpha + \frac{1}{3n}\right)^{n-1}\left(\alpha + (n-1)\frac{1}{3n}\right)\left(\alpha - (n-1)\frac{1}{3n}\right) = 0$$

spectrum is $Spec_{RS}(S_n^0)$ $=\begin{pmatrix} \frac{1}{3n}(n-1) & -\frac{1}{3n}(n-1) & -\frac{1}{3n} & \frac{1}{3n} \\ 1 & 1 & n-1 & n-1 \end{pmatrix}$.

Therefore, $RSE(S_n^0) = \frac{4(n-1)}{3n}$. □

Theorem 3. *The Randic status energy of* $K_{n\times 2}$ *is* $RSE(K_{n\times 2}) = \frac{n-1}{n}$.

Proof. Here the status of each vertex is $2n$. Hence Randic status matrix is

$$\begin{bmatrix} 0 & 0 & \frac{1}{2n} & \frac{1}{2n} & \cdots & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} \\ 0 & 0 & \frac{1}{2n} & \frac{1}{2n} & \cdots & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} \\ \frac{1}{2n} & \frac{1}{2n} & 0 & 0 & \cdots & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} \\ \frac{1}{2n} & \frac{1}{2n} & 0 & 0 & \cdots & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{2n} & \frac{1}{2n} & \cdots & \frac{1}{2n} & \frac{1}{2n} & 0 & 0 & \frac{1}{2n} & \frac{1}{2n} \\ \frac{1}{2n} & \frac{1}{2n} & \cdots & \frac{1}{2n} & \frac{1}{2n} & 0 & 0 & \frac{1}{2n} & \frac{1}{2n} \\ \frac{1}{2n} & \frac{1}{2n} & \cdots & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} & 0 & 0 \\ \frac{1}{2n} & \frac{1}{2n} & \cdots & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} & \frac{1}{2n} & 0 & 0 \end{bmatrix}.$$

$$Spec_{RS}(K_{n\times 2}) = \begin{pmatrix} \frac{1}{2n}(n-1) & 0 & -\frac{1}{2n} \\ 1 & n & n-1 \end{pmatrix}.$$

$$RSE(K_{n\times 2}) = \frac{n-1}{n}.$$

□

Theorem 4. *The Randic status energy of the complete bipartite graph* $K_{m,n}$ *is* $RSE(K_{m,n}) = 2\frac{\sqrt{mn}}{\sqrt{(n+2m-2)(m+2n-2)}}$.

Proof. Let $K_{m,n}$ be the complete bipartite graph of order $2n$ with vertex set $\{u_1, u_2, \cdots, u_m, v_1, v_2, \cdots, v_n\}$.
The Randic status matrix is

$$RS(K_{m,n}) = \frac{1}{\sqrt{(n+2m-2)(m+2n-2)}}\begin{bmatrix} 0 & 0 & 0 & \dots & 1 & 1 & 1 \\ 0 & 0 & 0 & \dots & 1 & 1 & 1 \\ 0 & 0 & 0 & \dots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 0 & 0 & 0 \\ 1 & 1 & 1 & \dots & 0 & 0 & 0 \\ 1 & 1 & 1 & \dots & 0 & 0 & 0 \end{bmatrix}.$$

So the characteristic equation is

$$\lambda^{m+n-2}\left(\lambda - \frac{\sqrt{mn}}{\sqrt{(n+2m-2)(m+2n-2)}}\right)\left(\lambda + \frac{\sqrt{mn}}{\sqrt{(n+2m-2)(m+2n-2)}}\right) = 0$$

and hence, the spectrum will be $Spec_{ABC}(K_{m,n}) = \begin{pmatrix} \frac{\sqrt{mn}}{\sqrt{(n+2m-2)(m+2n-2)}} & 0 & -\frac{\sqrt{mn}}{\sqrt{(n+2m-2)(m+2n-2)}} \\ 1 & m+n-2 & 1 \end{pmatrix}$.

Therefore,

$$RSE(K_{m,n}) = 2\frac{\sqrt{mn}}{\sqrt{(n+2m-2)(m+2n-2)}}.$$

□

III. Randic Status Energy of Complements

Theorem 12. *The Randic status energy of the complement $\overline{K_n}$ of the complete graph K_n is $LEDPE(\overline{K_n}) = 0$.*
Proof. Since all the entries of *Randic status matrix* are zero. The eigenvalues of the matrix are zero. Thus the *Randic status* energy of the complement of the complete graph K_n is

$$RSE(\overline{K_n}) = 0.$$

□

IV. Properties of Randic Status Energy

The following results will give the initial coefficients of the Randic status charecteristic polynomial and can be easily proven using the definition of characteristic polynomial.
Proposition 4. *In the Randic status charecteristic polynomial $\phi_{RS}(G, \alpha)$, the first three coefficients are* 1, 0 *and* $-\sum_{i=1}^{n}\left[\frac{1}{\sqrt{\sigma(u)\cdot\sigma(v)}}\right]^2$ *respectively.*
Proposition 5. $\sum_{i=1}^{n}\alpha_i{}^2 = 2\sum_{i=1}^{n}\left[\frac{1}{\sqrt{\sigma(u)\cdot\sigma(v)}}\right]^2$. *(where α_i represents Randic status eigenvalues.)*
The below theorems give the upper and lower bounds of the *Randic status energy*.

Theorem 6. *Let G be a graph with n vertices. Then* $RSE(G) \le \sqrt{2n\sum_{i=1}^{n}\left[\frac{1}{\sqrt{\sigma(u)\cdot\sigma(v)}}\right]^2}$.

Theorem 7. $RSE(G) \ge \sqrt{2\sum_{i=1}^{n}\left[\frac{1}{\sqrt{\sigma(u)\cdot\sigma(v)}}\right]^2 + n(n-1)\left[Det(RS(G))\right]^{\frac{2}{n}}}$.

References

[1] Gutman, The energy of a graph, Ber. Math. Stat. Sekt. Forschungsz. Graz, 103 (1978), 1-22.
[2] V. Lokesha, Y. Shanthakumari and P. Siva Kota Reddy, Skew-Zagreb Energy of Directed Graphs, Proceedings of the Jangjeon Mathematical Society, 23(4) (2020), 557-568.
[3] K. V. Madhusudhan, P. Siva Kota Reddy and K. R. Rajanna, Randic type Additive connectivity Energy of a Graph, Vladikavkaz Mathematical Journal, 21(2) (2019), 18-26.
[4] K. N. Prakasha, Randic status index, (To appear).
[5] K. N. Prakasha, P. Siva Kota Reddy and I. N. Cangul, Partition Laplacian Energy of a Graph, Advanced Studies in Contemporary Mathematics (Kyungshang), 27(4) (2017), 477-494.

[6] K. N. Prakasha, P. Siva Kota Reddy and I. N. Cangul, Symmetric division deg energy of a graph, Turkish Journal of Analysis and Number Theory, 5(6) (2017), 202-209.
[7] K. N. Prakasha, P. Siva Kota Reddy and I. N. Cangul, Minimum Covering Randic energy of a graph, Kyungpook Mathematical Journal, 57(4) (2017), 701-709.
[8] K. N. Prakasha, P. Siva Kota Reddy and I. N. Cangul, Sum-Connectivity Energy of Graphs, Advances in Mathematical Sciences and Applications, 28(1) (2019), 85-98.
[9] K. N. Prakasha, P. Siva Kota Reddy and I. N. Cangul, Atom-Bond-Connectivity Index of Certain Graphs, TWMS Journal of Applied and Engineering Mathematics, 13(2) (2023), 400-408.
[10] R. Rajendra, K. B. Mahesh and P. Siva Kota Reddy, Mahesh Inverse Tension Index for Graphs, Advances in Mathematics: Scientific Journal, 9(12) (2020), 10163--10170.
[11] R. Rajendra, P. Siva Kota Reddy and C. N. Harshavardhana, Tosha Index for Graphs, Proceedings of the Jangjeon Mathematical Society, 24(1) (2021), 141-147.
[12] R. Rajendra, P. Siva Kota Reddy and C. N. Harshavardhana, Rest of a vertex in a graph, Advances in Mathematics: Scientific Journal, 10(2) (2021), 697-704.
[13] R. Rajendra, K. B. Mahesh, and P. Siva Kota Reddy, Square Root Stress Sum Index for Graphs, Proyecciones, 40(4) (2021), 927-937.
[14] R. Rajendra, K. Bhargava, D. Shubhalakshmi and P. Siva Kota Reddy, Peripheral Harary Index of Graphs, Palestine Journal of Mathematics, 11(3) (2022), 323-336.
[15] R. Rajendra, P. Siva Kota Reddy, K. B. Mahesh and C. N. Harshavardhana, Richness of a Vertex in a Graph, South East Asian Journal of Mathematics and Mathematical Sciences, 18(2) (2022), 149-160.
[16] Ruby Merlin Pinto, R. Rajendra, P. Siva Kota Reddy and I. N. Cangul, A QSPR Analysis for Physical Properties of Lower Alkanes Involving Peripheral Wiener Index, Montes Taurus Journal of Pure and Applied Mathematics, 4(2) (2022), 81–85.
[17] R. Rajendra, P. Siva Kota Reddy and M. Prabhavathi, Computation of Wiener Index, Reciprocal Wiener index and Peripheral Wiener Index Using Adjacency Matrix, South East Asian Journal of Mathematics and Mathematical Sciences, 18(3) (2022), 275-282.
[18] R. Rajendra, P. Siva Kota Reddy, C. N. Harshavardhana, S. V. Aishwarya and B. M. Chandrashekara, Chelo Index for graphs, South East Asian Journal of Mathematics and Mathematical Sciences, 19(1) (2023), 175-188.
[19] R. Rajendra, P. Siva Kota Reddy, C. N. Harshavardhana and Khaled A. A. Alloush, Squares Stress Sum Index for Graphs, Proceedings of the Jangjeon Mathematical Society, 26(4) (2023), 483-493.
[20] R. Rajendra, P. Siva Kota Reddy and C. N. Harshavardhana, Stress-Difference Index for Graphs, Boletim da Sociedade Paranaense de Matemática (Terceira Série), 42 (2024), 1-10.