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Design and Performance Analysis of Digital FIR Filter based on PSO Algorithm using MATLAB

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***Abstract*—Filter has an important role in digital signal processing which is basically used in the applications for the minimization of distortion in the system. For transmitting and receiving of unaffected signal, the designing of filters is required, those eliminate the distortion from the desired signal efficiently and no noise be added in signal processing itself. The various important uses of the digital filter has strained the researchers, engineers, and scientists to design such filters with spontaneous, capable, rapid techniques by the usage of advanced technology with intelligent tools. F.I.R filters are more favored over IIR filters due to linearity in the phase response and frequency stability. Various windowing functions are existing i.e Hamming, Rectangular, Blackman, Hanning, Flat-top etc but the Kaiser-window function is more advantageous due to the variable parameters to control width of main lobe and to attenuate side band ripples. Various optimization algorithms may be utilized and the researchers analyze advantages and disadvantages. Particle Swarm Optimization algorithm(P.S.O) is become a known malleable method based on Swarm-Intelligence, mainly particle's population in search space. P.S.O enhance the features of result by renovate the position and velocity of swarms. Optimized group of filter coefficients have been generated using P.S.O algorithm that provide the improved result in the stop and pass band. Digital FIR filter has been fabricated in this paper by using Blackman, Flat-top and Kaiser window function. The designed filter i.e. FIR using Kaiser window function provides the improved results. PSO algorithm has been applied on designed filter to further optimize the design in MATLAB. Results obtained by using PSO show that devised F.I.R filter is superior than earlier devised F.I.R filter with reference to frequency spectrum.**

***Index Terms*— Finite Impulse Response (F.I.R) Filter, Particle Swarm Optimization(P.S.O), Swarm Intelligence (S.I), Blackman, Flat-top, Kaiser window Functions.**

I. Introduction

Digital signal processing (DSP) is most trending field, as the signal generation, its transmission and recovering the desired signal/audio/image/video is of prime importance. A lot of noises, interferences are generally mixed with the desired/ transmitted signal. It is the utmost priority to recover desired signals/images/videos. The filter is normally utilized to eliminate undesired distortion arise during operation of video, image and signal at any level i.e. sources, transmission and destination of communication system. Digital filter is beneficial and skilled for efficiently removing distortion from the distorted signal. FIR and IIR filters are the classification of digital

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filters [1-4]. Based upon dimension of information, classification of filter is one-dimensional (1D) for signal and two dimensional (2D) filters for image processing. Various windowing functions are existing but Kaiser Window function is more advantageous as contains variable parameters [5]. To solve these problems and to perform optimization process efficiently, signal processing requires a systematic and effective algorithm. Differential evolution[6] and genetic algorithm[7] are also popular multi-objective algorithms. Many optimization problems are solved by other algorithms like Artificial Bee Colony and ant colony optimization[8]. As various type of optimization problems can be interpreted by most of the meta-heuristic algorithms but they have many drawbacks too, such as many parameters used to be tuned, requirement of fine programming skills, high estimation cost and conversion of algorithms into binary forms. Swarm algorithms are used to interpret the optimization problems of real world. PSO is well known as well as among the most trusted swarm based algorithms[9]. The easy implementation, coding, limited controlling parameters and flexible & easy to hybridize with other optimization algorithms are the main reasons behind the popularity of PSO. Different performance may occur due to a minor alteration in controlling parameters. In PSO, set of swarm particles utilized and all particles keep their finest result in process of optimization. With right choice, PSO meets the optimized result proficiently.

Remaining paper is organized as follows: section II describes the literature review. Section III describes FIR filter design and mathematical representation. In section IV F.I.R filter design using P.S.O is discussed. Section V presents discussion and results. Section VI describes conclusions.

II. Literature Review

In this research paper, new techniques like subsection sampling and proposed FIR filter for the realization of the TDA function. This is to help in the extraction of required periodic components from a signal contain noise [10].

In this research paper, real time audio signal analysis and reduction of noise present in the information signal is performed using digital filtering techniques. Digital Finite Impulse Response filter, IIR filter, and Wavelet transform are employed [11].

In this research paper, various filtering techniques are utilized to minimize noise effects in ECG signal. FIR filter as LPF is presented, which helps to decrease attenuation in the ECG signal. FIR eliminates the high-frequency components and as LPF removes the noise at a low level from the ECG signal. Moving average filter (MAF) takes the average values of the signal and produces desired signal [12].

In this research paper, the uses of digital filters are elaborated for communication system. It is required to design an efficient digital FIR (finite impulse response) filter with proper window function. Window functions such as Blackman, Triangular, Rectangular, Kaiser, Hanning and Hamming are used for the designing of digital FIR filters [13].

In these research papers, reviews of research on Particle Swarm Optimization and specific applications are being discussed. [14], [15]

In this research paper, authors reviewed existing PSO variants. After reviewing latest papers and research on P.S.O, PSO is being considered in uninterrupted search space by authors. In this paper, many crucial attribute like PSO applications and binary form have not mentioned during optimization problems[16].

In this research paper, P.S.O parameters have been customized. P.S.O is appeared as excellent tool to optimize. Although, PSO undergo from premature convergence. Authors offered their work to use P.S.O for many technical/general outcomes. [17]

In this research paper, two dimensional filters using S.A & P.S.O is given by authors. Research is foundation upon integrating the limited search of S.A & universal search of P.S.O to control weaknesses of algorithm. [18]

In this research paper, devising of L.P.F.I.R filter with Kaiser window function has been projected by the authors. Authors used beta (β). Authors made the comparative reading of diverse order of filter to manage transition bandwidth. [19]

In this research paper, Particle Swarm Optimization (PSO) is used in designing of low pass FIR filter. The designing of filter depends on the excellent result found from the P.S.O for co-efficients. Velocity vectors have customized so particle vectors be enhanced results. These improved solutions develop the better result quality acquired by P.S.O algorithm. [20]

In this research paper, authors describe improvement of malleable equalizer using Q.P.S.O. Conduction of P.S.O has been compared to Constant Weight Inertia PSO, Least Mean Square (LMS) P.S.O algorithm to verify effectiveness of P.S.O. Authors observed that QPSO improves performance. [21]

In this research paper, survey of the linear phase L.P.F.I.R filter has been described. P.S.O algorithm & customized inertia-weight of P.S.O is used towards improvement of swarm's searching ability. Here, it is asserted, P.S.O simulation executes nice in comparison to remaining methods. Exact Coefficient and fast convergence can be achieved with P.S.O algorithm. [22]

In this research paper, PSO has been utilized to devise Low Pass F.I.R filter. By using PSO, excellent filter's coefficient has acquired. In this, writers claimed, "P.S.O is an easy swarm optimization method to optimize multidimensional difficult technical confront. [23]

In this research paper, filter has been devised & made comparison between different parameters & coefficients. Grasshopper Optimization algorithms (GOA) is taken to optimize the concert. L.P.L.P.F.I.R filter is devised by G.O.A. Design of L.P, H.P, B.P, and B.S filter has been described. Writers declared that ripples have been compacted the use of G.O.A. [24]

In this research paper, PSO has been used for the designing of FIR filter. The performance of A.R.P.S.O & C.R.P.S.O has been compared using MATLAB to design F.I.R filter. Performance has been matched to P.M. Acquired solutions shows that PSO-based algorithm performs better while designing linear phase FIR filters regarding RMS error & frequency spectrum. [25]

In this research paper, broad analysis of many optimization techniques for devising F.I.R & I.I.R has been obtained. Nature inspired methods to optimize difficult engineering issues have been discussed. Authors explained various optimization algorithms like Swarm Intelligence (SI), Genetic Algorithms (GA), Artificial Bee Colony (A.B.C), Particle Swarm Optimization (P.S.O), Grasshopper Optimization Algorithm (G.O.A), Cuckoo Search (C.S), Ant Colony Optimization (A.C.O), Biogeography-based optimization (B.B.O), Bacterial foraging optimization (B.F.O), Bat Algorithms (B.A)

III. Design of Finite Impulse Response Filter

Filter, whose impulse response is of finite time period is called finite impulse response (F.I.R) filter [28], [29]. Impulse response is finite as any feedback has not been provided to filter. Single dimensional digital F.I.R filter is illustrated by transfer function H(z) is:

$$\mathrm{H(Z)} = \sum_{n=0}^{N-1} \mathrm{h(n)} z^{-\mathrm{n}} \qquad (1)$$

$$\mathrm{H}(z) = □(0) + □(1)\mathrm{z}^{-1} + - - + \hbar\,(n)\mathrm{z}^{-(\mathrm{N}-1)} \qquad (2)$$

Where N is filter length of the impulse response $h(n)$. Output y(n) in time domain is

$$y(n) = x(n) * h(n) \qquad (3)$$

And output Y(z) in frequency domain is,

$$Y(\mathrm{z}) = X(\mathrm{z})\, H(\mathrm{z}) \qquad (4)$$

In equation (3) & (4), $x(n)$ & X(z) are inputs in time & frequency domain . Freq. response of single dimensional F.I.R filter is;

$$\mathrm{H}(w_k) = \Sigma\ \mathrm{h\ (n)}\ e^{-jw_k n} \qquad (5)$$

Where w_k = 2πk/n; (w_k) is F.T complex vector. Frequency [0, π] sampled to N values.

A. Blackman Window

Time domain sequence of the Blackman window is:

$$\mathrm{w(n)} = 0.42 - 0.5\cos\left(\frac{2\pi \mathrm{n}}{\mathrm{M}-1}\right) + 0.008\cos\left(\frac{4\pi \mathrm{n}}{\mathrm{M}-1}\right), \qquad 0 \le \mathrm{n} \le \mathrm{M}-1 \qquad (6)$$

Where, M is N/2 when N is even and (N+1)/2 when N is odd,

$\mathrm{M} \le n \le N - 1$, has achieved as reflecting 1st half around midpoint due to symmetry of Blackman. Symmetric method is a favorite technique while Blackman Window used to devise F.I.R filter.

Where M denotes length of a filter and n denotes time. Periodic Blackman Window designed using broadens required window length with single sample to N+1, designing symmetric window by removal of last sample. Favorite technique to use Blackman window in spectral analysis is periodic version as Discrete Fourier transform considers the periodic extension of an i/p vector.

B. Flat top Window

Flat top window is the summation of cosines. Co-efficients of flat top window are calculated by given equation:

$$w(n)=a_0 - a_1 cos\left(\frac{2\pi n}{N-1}\right) + a_2 cos\left(\frac{4\pi n}{N-1}\right) - a_3 cos\left(\frac{6\pi n}{N-1}\right) + a_4 cos\left(\frac{8\pi n}{N-1}\right) \quad (7)$$

Where $0 \leq n \leq N-1$. Values of co-efficients are

$a_0 = 0.2155789$, $a_1 = 0.4166316$, $a_2 = 0.27726316$, $a_3 = 0.8357895$, $a_4 = 0.00694737$

Flat-top window has low pass-band ripples (<0.01dB) and used mainly to calibrate function.

C. Kaiser window

It approximates prolate spheroidal window i.e. pointy instead of squashed. Basically, it is ratio of main-lobe to side-lobe energy. β manages relative side-lobe attenuation. Parameter β, side-lobe attenuation is rigid with respect to the window length.

Kaiser statement (n,beta) calculate the length n and parameter β. Co-efficients are calculated as

$$w(n) = \frac{I0\left(\beta\sqrt{1-\left(\frac{n-\frac{N}{2}}{\frac{N}{2}}\right)^2}\right)}{I0(\beta)} \quad \text{for } 0 \leq n \leq N \quad (8)$$

Where, I0 is zero order updated Bessel functions.

$$\text{length } L = N + 1 \quad (9)$$

Bessel i(0, beta * sqrt (1-(((0:L-1) - (L-1)/2) / ((L-1)/2)).^2)) / Bessel i(0, beta)

Kaiser Window has utilized for designing of F.I.R. filter. B is utilized to attain relative side lobe attenuation of α dB.

$$\beta = (0.1102\,(\alpha - 8.7)) \;; \quad \text{for } \alpha > 50 \quad (10)$$

$$\beta = 0.5842\,(\alpha - 21)^{0.4} + 0.07886(\alpha - 21) \quad for \;\; 50 \geq \alpha \geq 21 \quad (11)$$

$$\beta = 0\,; \;\; \text{for } \alpha < 21 \quad (12)$$

$$n = \frac{\alpha - 8}{2.285\Delta w} + 1 \quad (13)$$

IV. PSO ALGORITHM IN FIR FILTER DESIGN

P.S.O was introduced and used as continuous real valued algorithm [30]. Swarm of element revolves in D-dimensional search space to find an optimal solution. Particle (i) tends to store the current velocity vector

$$V_i = [v_{i1}, v_{i2}, \ldots, v_{iD}] \quad (14)$$

Current position vector stored as

$$X_i = [x_{i1}, x_{i2}, \ldots, x_{iD}] \quad (15)$$

P.S.O has started by taking arbitrarily X_i & V_i. After every iteration, particle attains excellent position i.e.

$$iPbest_i = [Pbest_{i1}, Pbest_{i2}, \ldots, Pbest_{iD}] \quad (16)$$

Finest position attained to swarms

$$Gbest = [Gbest_1, Gbest_2, \ldots, Gbest_D] \quad (17)$$

Updated position and velocity of particle is given below:

$$v_{id}\,(t+1)= [v_{id}\,(t)+c_1r_1(Pbest_{id}(t)-x_{id}(t))+c_2r_2(Gbest_d(t)-x_{id}(t))] \quad (18)$$

$$x_{id}(t+1)=x_{id}(t)+v_{id}\,(t+1) \quad (19)$$

V. RESULTS AND DISCUSSIONS

A F.I.R L.P.F is designed in MATLAB Simulink using DSP toolbox. The table-1given below illustrates the specified parameters utilized to devise digital F.I.R filter. Table includes structure type, response, designing method, filter order and frequency specification for digital filters. The FIR filter was designed using these parameters for both Blackman and Flat top windows. [31]. Magnitude response of the designed filter for both windows are revealed in diagrams (1 & 2) given below.

The table-2 given below describes the specified parameters utilized to devise digital F.I.R filter using Kaiser window. Table includes variable parameter beta, length of window function, max. & min. amplitude in time domain and main lobe width and side lobe attenuation in frequency domain for digital filters. [32]. Magnitude Response of devised filters for Kaiser window is given below in fig (3).

TABLE I-FIR FILTER PARAMETER SPECIFICATIONS FOR BLACKMAN AND FLAT TOP WINDOW

Filter Parameter	Value
Structure	Direct FIR Filter
Response	Low Pass Filter
Devise Method	Flat top and Blackman window
Filter order	34
Frequency Specification	(0-1) Normalized
Cut-off frequency	0.2

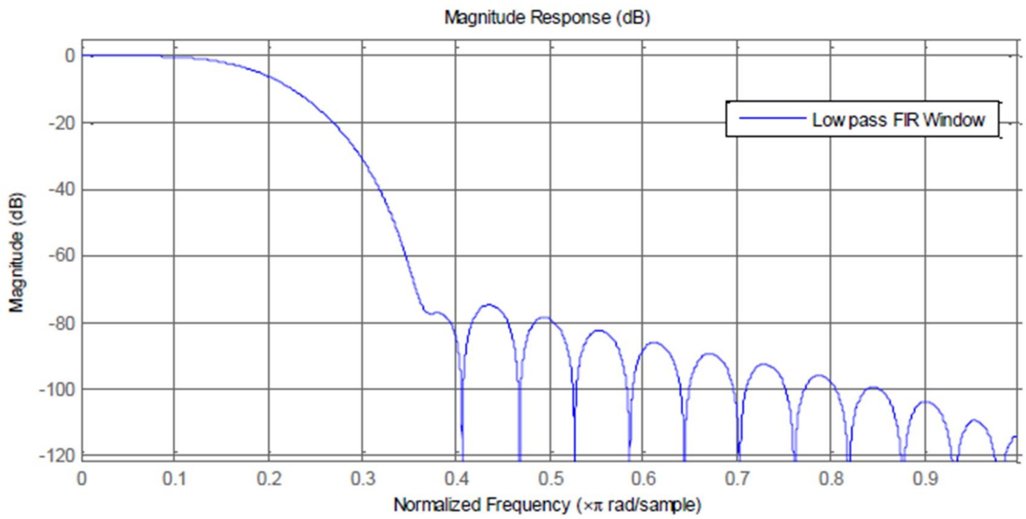

Fig.1 Magnitude Response (dB) of F.I.R for Blackman window

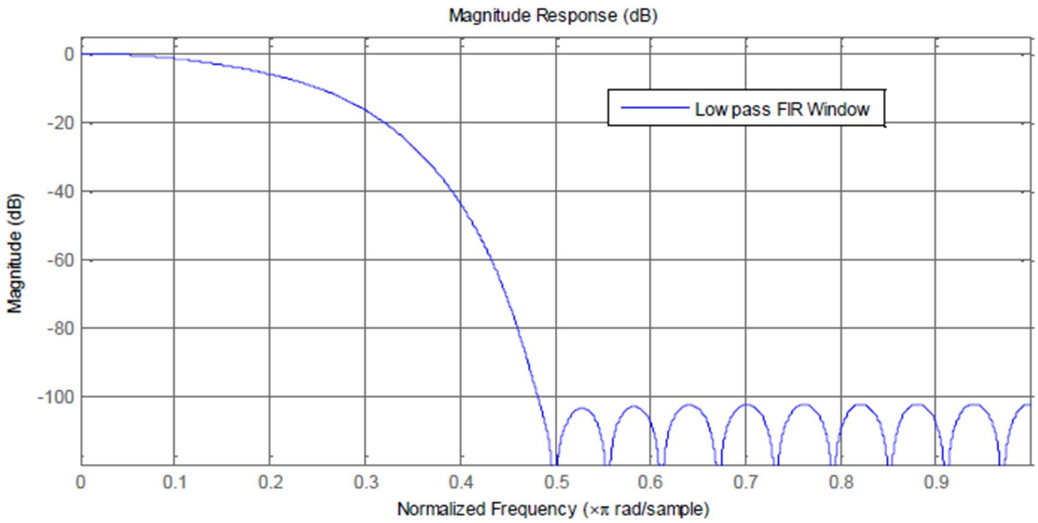

Fig.2 Magnitude Response of F.I.R for Flat-top window

TABLE II-FIR FILTER PARAMETER SPECIFICATIONS FOR KAISER WINDOW

Variable parameter beta	Window function's length	Time-domain		Frequency-domain	
		Maximum Amplitude	Minimum Amplitude	Main-lobe width	Relative side-lobe attenuation(dB)
4	38	1.0	0.01	0.625000	-31.0

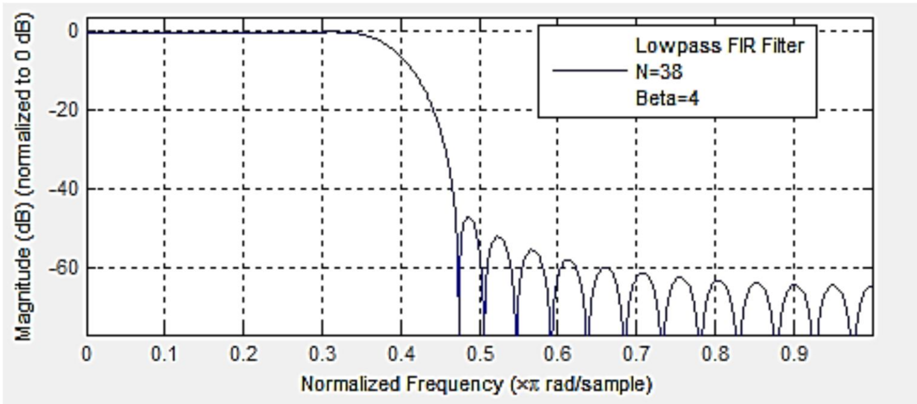

Fig.3 Mag. Response of Low pass F.I.R Filter

F.I.R filter utilizing Kaiser window function is more efficient. Main lobe's width and side-lobe's attenuation can be controlled by the beta parameters and the length (n). But it has a drawback too, i.e. when value of beta is increased, width main lobe's width increases & side lobe's attenuation decreases. [33]

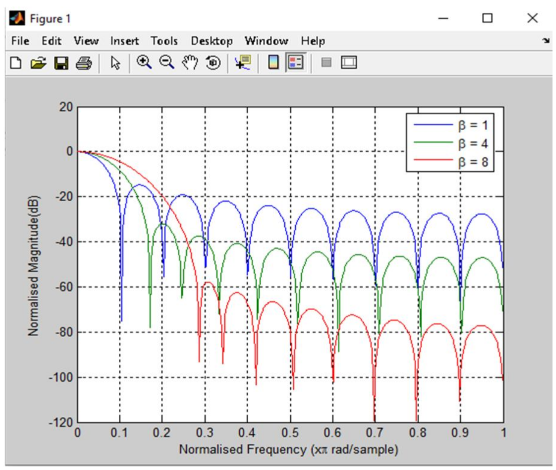

Fig.4: Kaiser Window of length 20 and different beta parameters

When the window size is increased, main lobe width and side lobe attenuation decreases shown in fig 5.

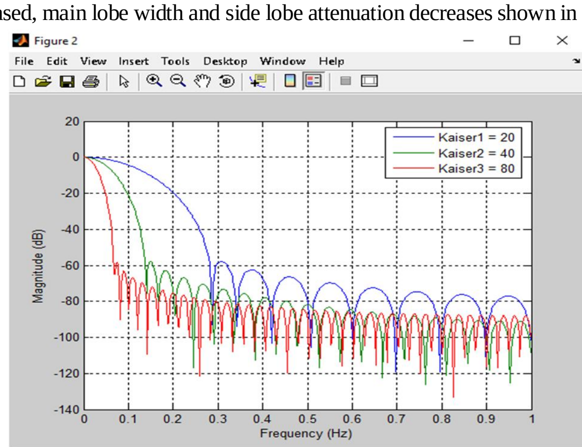

Fig.5: K. window of beta 8 and different lengths

TABLE III. PARAMETER FOR P.S.O

Parameters	PSO Value
Order of Filter	20.0
Size of Swarm	22.0
Acceleration Coef. (C1)	02
Acceleration Coef. (C2)	02
Inertia Coef. (w)	0.30
Damping Ratio of Inertia Coefficient (w damp)	0.99
Max no. of Iteration	100

P.S.O algorithms are executed on devised F.I.R filters utilizing the K.Window function of length n = 20 & β=8 which is common in above figures. P.S.O devises values given below Tab.3.
Implementation of provided values on Band Stop F.I.R filter provided following result.(Fig.6).

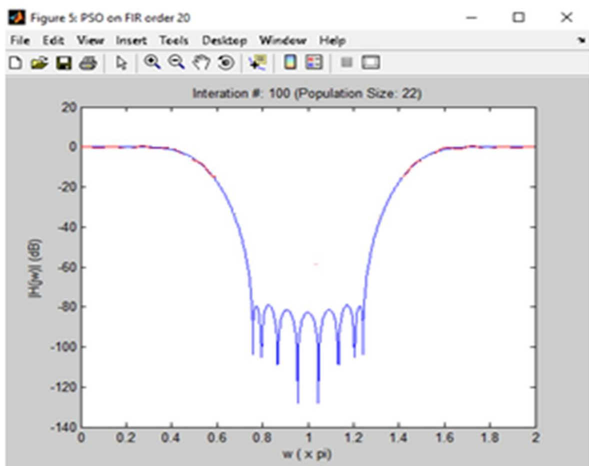

Fig. 6: O/P of P.S.O on 20th order F.I.R filter

VI. CONCLUSION

Digital F.I.R filter devised by the use of Blackman Window, Flattop window and Kaiser Window Function. Magnitude response of L.P. F.I.R filter is analyzed by given window-functions. FIR filter's magnitude response achieved nice by using Blackman window. It is observed that Flat-top window function performs better than Blackman Window function and performance is best using Kaiser Window function. Further, P.S.O method employed to devise the Band Stop F.I.R filters by the use of Kaiserwin. function. P.S.O modifies result due to an exclusive application offered to update position and velocity of swarm. A batch of upgraded coefficients have been acquired with P.S.O algorithm, gives advanced solutions to stop band. F.I.R filters have been designed utilizing Kaiser-window function in MATLAB efficiently. This devised filter is further upgraded by P.S.O algorithm. Solution shows, devised F.I.R filter performs excellent in comparison to earlier devised F.I.R filters. Stop band ripples decrease to a vast extent.

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