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A Functionally Graded Microelongated Layer Immersed in Non-Viscous Fluid

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***Abstract*—The present research article deals with two-dimensional deformation in a micro-ellongated functionally graded layer of finite thickness *2p* which is immersed in an infinite non-viscous fluid. A mechanical force of magnitude *P* is applied along the interface of two medium. The mechanical properties of functionally graded medium are taken to be in *x* direction. The analytical expressions for displacement, temperature, micro-elongation, stresses are obtained by using normal mode technique.**

***Index Terms*— Micro-elongation, Infinite Fluid, Functionally graded, Normal mode analysis.**

I. INTRODUCTION

To study the drawback of uncoupled theory of thermo-elasticity that the elastic deformation has no effect on temperature, Biot [1] developed the coupled theory of thermo-elasticity. In order to modify the uncoupled and coupled theory of thermo-elasticity, different researchers gave different theories of thermo-elasticity. The shortcomings of classical theory were removed by Eringen and Suhubi [2, 3] by modifying it into “Non linear theory of micro-elastic solids”. Later on, it was found that any micropolar thermoelastic material experiences macro deformation and micro rotation which was defined by Eringen [4, 5, 6] and named this theory as “linear theory of micro-polar elasticity”. Further Eringen [7] developed a theory named micropolar elastic solid with stretch in which he comprised axial stretch. Thermal effects in micro-polar theory were defined by different researcher [8, 9, 10]. The first theory of thermo-elasticity was generalized thermoelasticity and the second theory is temperature rate dependent theory which was given by Lord-Shulman [11]. Muller [12] defined an entropy production inequality in thermodynamics of thermo-elastic solids that helped him in considering the restriction on a class of constitutive equations. The generalized version of this inequality was presented by Green and Laws [13]. Another type of these constitutive equations was attained by Green and Lindsay [14]. The constitutive equations that contains two relaxation time constants was presented independently and explicitly by Suhubi [15]. The study of thermal and mechanical sources in generalised thermo-elastic half space was done by Sharma and Chauhan [16]. Sharma *et. al.* [17] investigated the steady-state response of an applied load moving with constant

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speed for infinite long time over the top surface of a homogeneous thermoelastic layer lying over an infinite half-space. A problem on laser pulse heating in thermo-elastic micro-elongated layer which is immersed in an infinite fluid was discussed by Ailawalia *et. al.* [21]. The study of wave propagation in thermo-elastic half space was done by Ailawalia and Sharma [22].

Functionally graded materials [23, 24, 25] which regulates the thermal variation was developed in Japan 1984 using powder metallurgy which makes FGM a significant thermal barrier. The thermo-mechanical analysis of FG cylinders and plates studied by Reddy and Chin [26]. Sankar and Tzeng [27] studied the thermal stresses in functionally graded beams. Abbas and Zenkour [28] studied the LS model on electro-magneto-thermoelastic response of an infinite functionally graded cylinder. Ghunghas *et. al.* [29] studied the influence of rotation and magnetic fields on a functionally graded thermoelastic solid. Kalkal *et al.*[30] discussed the two dimensional magneto thermoelastic interactions in a micropolar functionally graded solid. Hygro-thermoelastic nonlinear analysis of functionally graded porous composite thin and moderately thick shallow panels was discussed by Pajand and Masoodi [31]. The analysis of two dimensional functionally graded thermo-elastic solid was done by Sharma and Ailawalia [32].

In the present study, by taking micro-elongation effect and non-homogeneous medium, we developed a model for thermoelastic micro-elongated functionally graded layer of thickness *2p* with an overlying infinite non-viscous fluid. A force of magnitude *P* is applied along the interface of two medium. The mechanical properties of FGM are taken to be in *x* direction. The expressions for displacement, stresses, temperature and micro-elongation are obtained by using normal mode technique.

II. Basic Equations

The field equation of motion for a non-homogeneous, isotropic, micro-elongated, thermo-elastic solids in the absence of body forces are given by Shaw and Mukhopadhyay [19]:

$$\tau_{kl} = \lambda\delta_{kl}u_{r,r} + \mu\left(u_{k,l} + u_{l,k}\right) - \beta_0\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T\delta_{kl} + \lambda_0\delta_{kl}\phi, \tag{1}$$

$$m_k = a_0\phi_{,k}, \tag{2}$$

$$s - \tau = \lambda_0 u_{k,k} - \beta_1\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T - \lambda_1\phi, \tag{3}$$

$$q_{,k} = \frac{K^* T_{,k}}{T_0}, \tag{4}$$

According to [33, 34, 35] the field equation of motion for the displacement, microelongation and temperature and equation of heat conduction changes are:

$$(\lambda + \mu)u_{j,ij} + \mu u_{j,ij} - \beta_0\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T_{,i} + \lambda_0\phi_{,i} = \rho\ddot{u}_i, \tag{5}$$

$$a_0\phi_{,ii} + \beta_1\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T - \lambda_1\phi - \lambda_0 u_{j,j} = \frac{1}{2}\rho j_0\ddot{\phi} \tag{6}$$

$$K^* T_{,ii} - \rho C^*\left(1 + t_0\delta_{1k}\frac{\partial}{\partial t}\right)\dot{T} - \beta_0 T_0\left(1 + t_0\delta_{1k}\frac{\partial}{\partial t}\right)\dot{u}_{k,k} - \beta_1 T_0\ddot{\phi} = 0. \tag{7}$$

where $\beta_0 = (3\lambda+2\mu)\,\alpha_{t_1}$, $\beta_1 = (3\lambda+2\mu)\,\alpha_{t_2}$,

a_0, λ_0, λ_1 - micro-elongation constant, K^* - thermal conductivity, $\alpha_{t_1}, \alpha_{t_2}$- linear thermal expansion coefficient, T- temperature, T_0-reference temperature, C^* - specific heat at constant strain, ϕ - micro elongation scalar, j_0 - micro inertia, t_0, t_1 - thermal relaxation times, ρ - density of the micro elongated medium, u_i - displacement vector, σ_{kl} - stress tensor micro-elongational, m_k - the component of microstretch vector, *s* - component of stress tensor, q_k denotes heat flux.

The equation of motion in a non-viscous fluid is given by Ewing *et. al.* [36]

$$\lambda^F u^F_{j,ij} = \rho^F \ddot{u}^F_i, \tag{8}$$

The relation of stress and displacement is given as

$$t^F_{ij} = \lambda^F u^F_{r,r}\delta_{ij} \tag{9}$$

Where λ^F denoted the fluid constant and ρ^F denoted the density of fluid.

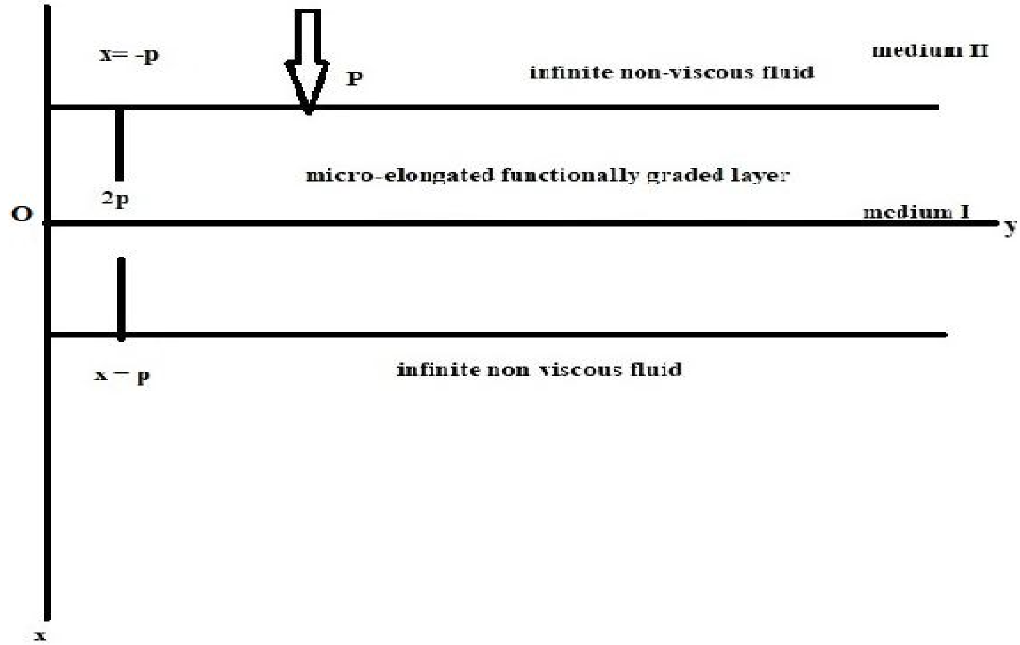
III. Formulation of the Problem

Here, we consider a rectangular cartesian co-ordinate system *(x, y, z)* with x axis pointing vertically downward. A functionally graded isotropic thermoelastic micro-elongated layer of finite thickness *2p* occupying the region $-p \le x \le p$ (medium I) having a displacement vector $\vec{u}$ given by $\vec{u} = (u, v, 0)$ where, *u = (x, y, t), v = (x, y, t)* and a displacement vector $\overrightarrow{u^F}$ with component $\overrightarrow{u^F} = (u^F, v^F, 0)$ for infinite non-viscous fluid (medium II) is considered.

Here we restricted our present study to x − y plane. Therefore, all the field quantities will not depend on the space variable *z.*

Due to the effect of functionally graded solid, the parameters are no longer constant but they depend on space variables. Here β_1, β_0, K^*, ρ, λ_1, λ_0, a_0, λ, μ are replaced by $\beta_{10}\,\psi(X)$, $\beta_{00}\,\psi(X)$, $K_0^*\,\psi(X)$, $\rho_0\,\psi(X)$, $\lambda_{10}\,\psi(X)$, $\lambda_{00}\,\psi(X)$, $a_{00}\,\psi(X)$, $\lambda_0\,\psi(X)$, $\mu_0\psi(X)$ respectively, where β_{10} , β_{00}, K_0^*, ρ_0, λ_{10}, λ_{00}, a_{00}, λ_0, μ_0 are constants.

$\psi(X)$is a given dimensionless function of space variable X = (x, y, z). We suppose that the properties of the medium depends on x coordinate only, hence $\psi(X)$ will be further expressed as $\psi(X)$.



Therefore field equation (5)–(7) becomes:

$$\psi(X)\left[(\lambda_0+2\mu_0)\frac{\partial^2 u}{\partial x^2}+(\lambda_0+\mu_0)\frac{\partial^2 v}{\partial x\partial y}+\mu_0\frac{\partial^2 u}{\partial y^2}-\beta_{00}\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}+\lambda_{00}\frac{\partial \phi}{\partial x}\right]+\frac{\partial}{\partial x}\psi(X)[(\lambda_0+2\mu_0)\frac{\partial u}{\partial x}+\lambda_0\frac{\partial v}{\partial y}-\beta_{00}\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\lambda_{00}\phi=\rho_0 f(x)\frac{\partial^2 u}{\partial t^2}, \tag{10}$$

$$\mu_0\psi(X)\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial x^2}\right)+\mu_0\frac{\partial}{\partial x}\psi(X)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\psi(X)(\lambda_0\frac{\partial^2 u}{\partial x\partial y}+(\lambda_0+2\mu_0)\frac{\partial^2 v}{\partial y^2}-\beta_{00}\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}+\lambda_{00}\frac{\partial \phi}{\partial y}=\rho_0 f(x)\frac{\partial^2 v}{\partial t^2}, \tag{11}$$

$$a_{00}\left[\psi(X)\nabla^2\phi+\frac{\partial}{\partial x}\psi(X)\frac{\partial \phi}{\partial x}\right]+\psi(X)\beta_{10}f(x)\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\mathrm{T}-\lambda_{10}\psi(X)\phi-\lambda_{00}\psi(X)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\frac{1}{2}\rho_0\psi(X)\frac{\partial^2\phi}{\partial t^2}, \tag{12}$$

$$K_0^*\left[\psi(X)\nabla^2 T+\frac{\partial}{\partial x}\psi(X)\frac{\partial T}{\partial x}\right]-\rho_0\psi(X)C^*\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}-\beta_{00}\psi(X)T_0\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-\beta_{10}\psi(X)T_0\frac{\partial \phi}{\partial t}=0, \tag{13}$$

The stress components are as,

$$\tau_{xx}=\psi(X)\left[(\lambda_0+2\mu_0)\frac{\partial u}{\partial x}+\lambda_0\frac{\partial v}{\partial y}-\beta_{00}\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\lambda_{00}\phi\right], \tag{14}$$

$$\tau_{yy}=\psi(X)\left[\lambda_0\frac{\partial u}{\partial x}+(\lambda_0+2\mu_0)\frac{\partial v}{\partial y}-\beta_{00}\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\lambda_{00}\phi\right], \tag{15}$$

$$\tau_{xy}=\mu_0\psi(X)\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right). \tag{16}$$

IV. Exponential Variation of Non-homogeneity

It is assumed that $\psi(X)=e^{-nx}$, where n is a dimensionless parameter, it can be concluded that material having mechanical as well as thermal properties alter exponentially along the x-direction.

The governing equation (10) – (13) and the stress equations (14) – (16) can be changed in the non-dimensional form by introducing the dimensionless
parameters:

$$x' = \frac{\omega^*}{c_1}x,\ y' = \frac{\omega^*}{c_1}y,\ t_0' = \omega^* t_0,\ T' = \frac{T}{T_0},\ t' = \omega^* t,\ u_i' = \frac{\omega^*\rho_0 c_1}{\beta_{00}T_0}u_i, \phi' = \frac{\lambda_{00}}{\beta_{00}T_0}\phi, \sigma_{ij}' = \frac{\sigma_{ij}}{\beta_{00}T_0}$$

where,

$$\omega^* = \frac{\rho_0 c_1^2 C^*}{K^*}\ c_1^2 = \frac{\lambda_0 + 2\mu_0}{\rho_0}$$

Using the above mentioned dimensionless parameters in equations (10)-(16), after dropping subscript we get:

$$[\frac{\partial^2 u}{\partial x^2} + l_2\frac{\partial^2 v}{\partial x \partial y} + b_3\frac{\partial^2 u}{\partial y^2} - (1+t_1\omega^*\delta_{2k}\frac{\partial}{\partial t})\frac{\partial T}{\partial x} + \frac{\partial \phi}{\partial x}] - n\,[\frac{\partial u}{\partial x} + l_4\frac{\partial v}{\partial y} - (1+t_1\omega^*\delta_{2k}\frac{\partial}{\partial t})T + \phi] = \frac{\partial^2 u}{\partial t^2}, \tag{17}$$

$$[b_3\frac{\partial^2 v}{\partial x^2} + b_2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} - (1+t_1\omega^*\delta_{2k}\frac{\partial}{\partial t})\frac{\partial T}{\partial y} + \frac{\partial \phi}{\partial y}] - n\, b_3[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}] = \frac{\partial^2 v}{\partial t^2}, \tag{18}$$

$$[\nabla^2\phi - n\frac{\partial \phi}{\partial x}] + b_5\,(1+t_1\omega^*\delta_{2k}\frac{\partial}{\partial t})\,T - b_6\phi - b_7\,(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = b_8\frac{\partial^2 \phi}{\partial t^2}, \tag{19}$$

$$[\nabla^2 T - n\frac{\partial T}{\partial x}] - b_9\,(1+t_0\omega^*\delta_{1k}\frac{\partial}{\partial t})\frac{\partial T}{\partial t} - b_{10}\,(\frac{\partial}{\partial t} + t_0\omega^*\delta_{1k}\frac{\partial^2}{\partial t^2})(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) - b_{11}\frac{\partial \phi}{\partial t} = 0\,, \tag{20}$$

$$\sigma_{xx} = [\frac{\partial u}{\partial x} + b_4\frac{\partial v}{\partial y} - (1+t_1\omega^*\delta_{2k}\frac{\partial}{\partial t})T + \phi]\, e^{-nx}, \tag{21}$$

$$\sigma_{yy} = [b_4\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - (1+t_1\omega^*\delta_{2k}\frac{\partial}{\partial t})T + \phi]\, e^{-nx}, \tag{22}$$

$$\sigma_{xy} = l_3(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})\, e^{-nx}, \tag{23}$$

where,

$b_2 = \frac{\lambda_0+\mu_0}{\rho_0 c_1^2}$, $b_3 = \frac{\mu_0}{\rho_0 c_1^2}$, $b_4 = \frac{\lambda_0}{\rho_0 c_1^2}$, $b_5 = \frac{\beta_{10}\lambda_{00}c_1^2}{\beta_{00}\omega^{*2}}$, $b_6 = \frac{\lambda_{10}c_1^2}{\omega^{*2}}$, $b_7 = \frac{\lambda_{00}^2}{\rho_0\omega^{*2}}$, $b_8 = \frac{1}{2}\rho_0 j_0 c_1^2$,
$b_9 = \frac{c^* c_1^2}{K_0^*\omega^*}$, $b_{10} = \frac{\beta_{00}^2 T_0}{\rho_0 K_0^*\omega^{*2}}$, $b_{11} = \frac{\beta_{10}\beta_{00}T_0 c_1^2}{\lambda_{00}K_0^*\omega^*}$.

V. Method of Solution

To obtain the analytical expressions for displacement, temperature, stresses
and micro-elongation, normal mode technique is used here. In this method, the solution of the physical variables is decomposed in terms of normal mode.

$$(u, v, T, \phi, \tau_{ij})(x, y, t) = (u^*, v^*, T^*, \phi^*, \tau_{ij}^*)(x)e^{\omega t + \iota b y} \tag{24}$$

where ω specifies complex frequency, b represents as the wave number, and u, v, T^*, $\phi^*$and τ_{ij}^* are field quantities amplitude.
Using (24) in (17)-(23) equations, we get

$$(D^2 - nD - g_1)u^* + \iota b(b_2 D - nb_4)v^* - g_2(D-n)T^* + (D-n)\phi^* = 0, \tag{25}$$

$$\iota b(b_2 D - nb_3)u^* + (b_3 D^2 - nb_3 D - g_3)v^* - g_2\iota b T^* + \iota b\phi^* = 0, \tag{26}$$

$$-b_7 D u^* - b_7\iota b v^* + b_5 g_2 T^* + (D^2 - nD - g_4)\phi^* = 0, \tag{27}$$

$$-b_{10}g_{15}Du^* - \iota b b_{10}g_5 v^* + (D^2 - nD - g_6)T^* - b_{11}\omega\phi^* = 0, \tag{28}$$

$$\tau_{xx} = Du^* + b_4\iota b v^* - g_2 T^* + \phi^* \tag{29}$$

$$\tau_{yy} = b_4 D u^* + \iota b v^* - g_2 T^* + \phi^* \tag{30}$$

$$\tau_{xy} = b_3(\iota b u^* + D v^*) \tag{31}$$

where,

$D = \frac{d}{dx}$, $b_1 = b_3 b^2 + \omega^2$, $b_{12} = (1 + t_1 \delta_{2k} \omega \omega^*)$, $b_3 = b^2 + \omega^2$, $b_4 = b^2 + b_6 + b_8 \omega^2$, $g_5 = \omega(1 + t_1 \delta_{2k} \omega \omega^*)$, $g_6 = b^2 + b_9(1 + t_0 \delta_{1k} \omega \omega^*)$.

After eliminating v∗, T∗, ϕ∗ from equations (25)-(28), we get eighth order differential equation for u∗ as:

$$(D^8 + nN_1 D^7 + N_2 D^6 + nN_3 D^5 + N_4 D^4 + nN_5 D^3 + N_6 D^2 + nN_7 D + N_8) u^*(x) = 0 \qquad (32)$$

where,

$N_1 = \frac{-(s_2 s_{12} + s_1 s_{12} - s_{16} b_3 + 2 s_{15} b_3)}{b_3 b_{11} \omega}$,

$N_2 = \frac{-(s_3 s_{12} - n^2 s_2 s_{12} + s_1 s_{21} - b_3 s_{17} - 2n^2 s_{16} b_3 - s_{15} s_4)}{b_3 b_{11} \omega}$,

$N_3 = \frac{-(-s_3 s_{12} - s_2 s_{21} + s_1 s_{22} + b_3 s_{18} - 2 s_{17} b_3 + s_{16} s_4 - s_5 s_{15})}{b_3 b_{11} \omega}$,

$N_4 = \frac{-(-s_3 s_{21} - n^2 s_2 s_{22} + s_1 s_{22} + b_3 s_{19} - 2n^2 s_{18} b_3 + s_{17} s_4 + n^2 s_5 s_{16})}{b_3 b_{11} \omega}$,

$N_5 = \frac{-(-s_3 s_{22} - s_2 s_{23} + s_1 s_{24} + l_3 s_{20} - 2 s_{19} l_3 + s_{18} s_4 + s_5 s_{17})}{b_3 b_{11} \omega}$,

$N_6 = \frac{-(s_{23} - n^2 s_2 s_{24} + s_1 s_{25} - 2n^2 s_{20} l_3 + s_{19} s_4 + n^2 s_5 s_{18})}{b_3 b_{11} \omega}$,

$N_7 = \frac{-(-s_3 s_{24} - s_{25} s_2 + s_{20} s_4 + s_5 s_{19})}{b_3 b_{11} \omega}$,

$N_8 = \frac{-(-s_3 s_{25} + n^2 s_5 s_{20})}{b_3 b_{11} \omega}$,

And

$s_1 = 1 - b_2$, $s_2 = 1 - b_2 + b_3$, $s_3 = g_1 + n^2 b_3$, $s_4 = b_2 b^2 - g_3 + n^2 b_3$, $s_5 = g_3 - b^2 b_4$, $s_6 = b_{10} g_5$,
$s_7 = b_{10} g_5 g_4 - b_7 b_{11} \omega$, $s_8 = n^2 - g_6 - g_4$, $s_9 = g_6 + g_4$,
$s_{10} = b_5 b_{11} \omega g_2 + g_4 g_6$, $s_{11} = b_2 b_{11} \omega - b_{10} g_5$, $s_{12} = b_3 b_{11} \omega$,
$s_{13} = b^2 b_{10} g_5 - g_3 b_{11} \omega$, $s_{14} = g_2 b_{11} \omega + g_6$, $s_{15} = b_2 b_{11} \omega$, $s_{16} = b_3 b_{11} \omega + 2 b_2 b_{11} \omega$,
$s_{17} = g_2 g_5 b_{10} b_{11} \omega - b_7 b_{11} \omega - 2n^2 b_3 b_{11} \omega - (n^2 - g_6 - g_4) b_2 b_{11} \omega$,
$s_{18} = -s_7 - s_6 s_{14} + s_{12} s_8 - s_9 s_{11}$, $s_{19} = s_7 s_{14} + n^2 s_9 s_{12} - s_{10} s_{11}$, $s_{20} = s_{10} s_{12}$,
$s_{21} = b^2 s_6 - s_{13} - 2n^2 s_{12} - s_8 s_{12}$, $s_{22} = -2b^2 s_6 + 2 s_{13} + s_{12} s_8 - s_{12} s_9$,
$s_{23} = b^2 s_7 + b^2 n^2 s_6 - s_6 b^2 s_{14} - s_8 s_{13} + n^2 s_{12} s_9 - s_{10} s_{12}$
, $s_{24} = b^2 s_7 + b^2 s_6 s_{14} - s_{13} s_{19} + s_{10} s_{12}$ $s_{25} = -s_7 b^2 s_{14} - s_{10} s_{13}$, $s_{26} = -s_1 s_{12} - s_{15} b_3$,
$s_{27} = s_{12}(s_2 + s_1) + b_3(s_{16} + 2 s_{15})$, $s_{28} = s_{12}(s_3 - n^2 s_2) + s_1 s_{21} + b_3(s_{17} - 2n^2 s_{16}) - s_{15} s_4$
$s_{29} = s_1 s_{22} + b_3(s_{19} - 2n^2 s_{18}) - s_3 s_{12} - s_2 s_{21} + s_4 s_{16} - s_{15} s_4$
$s_{30} = s_1 s_{22} + b_3(s_{19} - 2n^2 s_{18}) - s_3 s_{21} - n^2 s_2 s_{22} + s_4 s_{17} + n^2 s_{16} s_5$
$s_{31} = s_1 s_{24} + s_{20} b_3 - s_3 s_{22} - s_2 s_{23} - 2 b_3 s_{19} + s_4 s_{18} - s_5 s_{17}$
$s_{32} = s_1 s_{25} - s_3 s_{23} - n^2 s_2 s_{24} - 2n^2 b_3 s_{20} + s_4 s_{19} + n^2 s_5 s_{18}$
$s_{33} = s_4 s_{20} - s_3 s_{24} - s_2 s_{25} - s_5 s_{19}$, $s_{34} = n^2 s_{20} s_5 - s_3 s_{25}$

The solution of equation (32) which is bounded as $x \to \infty$ is given by

$$u^*(x) = \sum_{i=1}^4 M_i(a,\omega) e^{-k_i x} + \sum_{i=1}^4 M_i'(a,\omega) e^{k_i x}, \qquad (33)$$

$$v^*(x) = \sum_{i=1}^4 R_{1i} M_i(a,\omega) e^{-k_i x} + \sum_{i=1}^4 R_{1i}' M_i'(a,\omega), \qquad (34)$$

$$T^*(x) = \sum_{i=1}^4 R_{2i} M_i(a,\omega) e^{-k_i x} + \sum_{i=1}^4 R_{2i}' M_i'(a,\omega) e^{k_i x}, \qquad (35)$$

$$\phi^*(x) = \sum_{i=1}^4 R_{3i} M_i(a,\omega) e^{-k_i x} + \sum_{i=1}^4 R_{3i}' M_i'(a,\omega) e^{k_i x}, \qquad (36)$$

where $\pm k_i (i = 1,2,3,4)$ are the roots of the equation (32) and $M_i(a,\omega), M_i'(a,\omega)$ $(i = 1,2,3,4)$ are the parameters, depending upon a and ω,

$$R_{1i} = \left[\frac{\iota b(s_1 k_i^2 + n s_2 k_i - s_3)}{(-b_3 k_i^3 - 2nb_3 k_i^2 - s_4 k_i + n s_5)}\right], \quad R_{1i}' = \left[\frac{\iota b(s_1 k_i^2 - n s_2 k_i - s_3)}{(b_3 k_i^3 - 2nb_3 k_i^2 + s_4 k_i + n s_5)}\right],$$

$$R_{2i} = \left[\frac{\iota b(s_{11} k_i + n s_{12}) - R_{1i}(s_{12} k_i^2 + n s_{12} k_i + s_{13})}{(\iota b k_i^2 + \iota b n k_i - \iota b s_{14})}\right], R_{2i}' = \left[\frac{-\iota b(s_{11} k_i - n s_{12}) - R_{1i}'(s_{12} k_i^2 - n s_{12} k_i + s_{13})}{(\iota b k_i^2 - \iota b n k_i - \iota b s_{14})}\right],$$

$R_{3i} = \frac{-b_7 k_i + b_7 \iota b R_{1i} - b_5 g_2 R_{2i}}{(k_i^2 + n k_i - g_4)}$, $\quad R'_{3i} = \frac{b_7 k_i + b_7 \iota b R'_{1i} - b_5 g_2 R'_{2i}}{(k_i^2 - n k_i - g_4)}$

Normal mode analysis of the stress components yields the following:

$$\sigma^*_{xx} = e^{-nx}[\sum_{i=1}^{4} M_i(a,\omega) e^{-k_i x} R_{4i} + \sum_{i=1}^{4} M'_i(a,\omega) e^{k_i x} R'_{4i}], \tag{37}$$

$$\sigma^*_{xy} = e^{-nx}[\sum_{i=1}^{4} M_i(a,\omega) e^{-k_i x} R_{5i} + \sum_{i=1}^{4} M'_i(a,\omega) e^{k_i x} R'_{5i}], \tag{38}$$

In the same way, the solution for infinite non-viscous fluid medium II are of the form

$$u^{F^*}(x) = M_5(a,\omega) e^{-k_5 x} + M'_5(a,\omega) e^{k_5 x}, \tag{39}$$

$$v^{F^*}(x) = R_{61} M_5(a,\omega) e^{-k_5 x} + R'_{61} M'_5(a,\omega) e^{k_5 x}, \tag{40}$$

The normal stress component is given by

$$\tau_{xx}{}^{F^*}(x) = R_{71} M_5(a,\omega) e^{-k_5 x} + R'_{71} M'_5(a,\omega) e^{k_5 x}, \tag{41}$$

where $M_5(a,\omega)$, $M'_5(a,\omega)$ are the specific functions depending upon a, ω and k_5^2 are the roots of the following second-degree equation

$$(D^2 - k_5^2) u^{f^*}(x) = 0, \tag{42}$$

Where,

$$k_5^2 = \frac{b_{12} b_{13} \omega^2}{b_{13} - b^2}.$$

$$b_{13} = \frac{\rho^F c_1^2}{\lambda^F}, \; b_{12} = b^2 + b_{12}\omega^2,$$

And

$$R_{61} = \frac{\iota b k_5}{b_{13}}, \; R'_{61} = -\frac{\iota b k_5}{b_{13}},$$

$$R_{71} = \frac{(-k_5 - \iota b R_{61})\lambda^F}{\beta_{00} T_0}, \qquad R'_{71} = \frac{(-k_5 - \iota b R_{61})\lambda^F}{\beta_{00} T_0}.$$

VI. BOUNDARY CONDITIONS

In this section, we will discuss the theoretical results to obtain the values of parameters M_i, $M'_i (i = 1,2,3,4)$, M_5, M'_5. A mechanical force of magnitude *P* is acting along the microelongated functionally graded layer of thickness *2p*. To find the values of the parameters M_i, $M'_i (i = 1,2,3,4)$, M_5, M'_5 the following boundary conditions are taken:

$$a)\; (\tau_{xx})_I = (\tau_{xx})_{II} - P e^{\omega t + \iota b y} \;\; at \;\; x = -p \tag{43}$$

$$b)\; (\tau_{xy})_I = 0 \;\; at \;\; x = \pm p \tag{44}$$

$$c)\; \frac{\partial T}{\partial x} = 0 \;\; at \;\; x = \pm p \tag{45}$$

$$d)\; (u)_I = (u)_{II} \;\; at \;\; x = \pm p \tag{46}$$

$$e)\; (v)_I = (v)_{II} \;\; at \;\; x = \pm p \tag{47}$$

$$f)\; (\tau_{xx})_I = 0 \;\; at \;\; x = p \tag{48}$$

where *P* is the magnitude of the constant force applied to the boundary.

Using equations (33)-(35), (37), (38) and (41), in the boundary conditions (43)-(48), we get ten equations in ten unknowns as:

$$\sum_{i=1}^{4} M_i(a,\omega) e^{(k_i+n)p} R_{4i} + \sum_{i=1}^{4} M'_i(a,\omega) e^{(n-k_i)p} R'_{4i} - R_{71} M_5 e^{k_5 p} - R'_{71} M'_5 e^{-k_5 p} = -P, \tag{49}$$

$$\sum_{i=1}^{4} M_i(a,\omega) e^{(-k_i-n)p} R_{5i} + \sum_{i=1}^{4} M'_i(a,\omega) e^{(k_i-n)p} R'_{5i} = 0, \tag{50}$$

$$\sum_{i=1}^{4} M_i(a,\omega)e^{(k_i+n)p}R_{5i} + \sum_{i=1}^{4} M_i'(a,\omega)e^{(-k_i+n)p}R_{5i}' = 0, \quad (51)$$

$$\sum_{i=1}^{4} -k_i R_{2i} M_i(a,\omega)e^{-k_i p} + \sum_{i=1}^{4} k_i R_{2i}' M_i'(a,\omega)e^{k_i p} = 0, \quad (52)$$

$$\sum_{i=1}^{4} -k_i R_{2i} M_i(a,\omega)e^{k_i p} + \sum_{i=1}^{4} k_i R_{2i}' M_i'(a,\omega)e^{-k_i p} = 0, \quad (53)$$

$$\sum_{i=1}^{4} M_i(a,\omega)e^{-k_i p} + \sum_{i=1}^{4} M_i(a,\omega)e^{k_i p} - M_5 e^{-k_5 p} - M_5' e^{k_5 p} = 0, \quad (54)$$

$$\sum_{i=1}^{4} M_i(a,\omega)e^{k_i p} + \sum_{i=1}^{4} M_i(a,\omega)e^{-k_i p} - M_5 e^{k_5 p} - M_5' e^{-k_5 p} = 0 \quad (55)$$

$$\sum_{i=1}^{4}(R_{1i}M_i(a,\omega)e^{-k_i p} + M_i'(a,\omega)e^{k_i p}R_{1i}') - (R_{61}M_5(a,\omega)e^{-k_5 p} + M_5'(a,\omega)e^{-k_5 p}R_{61}' = 0 \quad (56)$$

$$\sum_{i=1}^{4}(R_{1i}M_i(a,\omega)e^{k_i p} + M_i'(a,\omega)e^{-k_i p}R_{1i}') - (R_{61}M_5(a,\omega)e^{k_5 p} + M_5'(a,\omega)e^{k_5 p}R_{61}' = 0 \quad (57)$$

To find the values of the constants $M_i\,, M_i'(i = 1,2,3,4)$, M_5, M_5' the system of equations from (49)-(57) evaluated using MATLAB.

Hence we obtain the components of displacement, temperature distribution, micro-elongation stresses as

$$u^*(x) = \frac{1}{\Delta}[\sum_{i=1}^{4} \Delta_i e^{-k_i x} + \sum_{i=1}^{4} \Delta_i' e^{k_i x}], \quad (58)$$

$$v^*(x) = \frac{1}{\Delta}[\sum_{i=1}^{4} R_{1i}\Delta_i e^{-k_i x} + \sum_{i=1}^{4} R_{1i}'\Delta_i' e^{k_i x}] \quad (59)$$

$$T^*(x) = \frac{1}{\Delta}[\sum_{i=1}^{4} R_{2i}\Delta_i e^{-k_i x} + \sum_{i=1}^{4} R_{2i}'\Delta_i' e^{k_i x}], \quad (60)$$

$$\phi^*(x) = \frac{1}{\Delta}[\sum_{i=1}^{4} R_{3i}\Delta_i e^{-k_i x} + \sum_{i=1}^{4} R_{3i}'\Delta_i' e^{k_i x}], \quad (61)$$

$$\tau_{xx}^* = e^{-nx}[\frac{1}{\Delta}(\sum_{i=1}^{4} \Delta_i e^{-k_i x}R_{4i} + \sum_{i=1}^{4} \Delta_i' e^{k_i x}R_{4i}')], \quad (62)$$

$$\tau_{xy}^* = e^{-nx}[\frac{1}{\Delta}(\sum_{i=1}^{4} \Delta_i e^{-k_i x}R_{5i} + \sum_{i=1}^{4} \Delta_i' e^{k_i x}R_{5i}')], \quad (63)$$

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