

Reduced Sombor Index of Certain Graphs

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Abstract—In 21st century, topological indices allured the mathematicians as well as chemists. Reduced Sombor index is one of the degree based molecular descriptor. In this paper, we elucidate the Reduced Sombor index for some graph structures such as double graphs, subdivision graphs, complements and chemical structures.

Index Terms— Reduced Sombor index, double graphs, subdivision graphs.

I. INTRODUCTION

21st century witnessed a rapid growth in the research related to molecular descriptors. Many new topological indices were introduced, and lot of research papers published (See for example [1-17]. Numerous applications identified on the topological indices in Chemistry and other allied fields. Ivan Gutman [3] introduced the Reduced Sombor index in chemical graph theory, which is defined as

$$RS(G) = \sum_{v_i \sim v_j} \sqrt{(d_i - 1)^2 + (d_j - 1)^2}.$$

Here we give few definitions for different graph structures which we used in the later sections.

Definition 1.1 The complement of a graph G is a graph $\bar{G}$ having the same vertex set such that two distinct vertices of $\bar{G}$ are adjacent if and only if they are not adjacent in G .

Definition 1.2. [18] The subgraph or subdivision graph $S(G)$ of a simple graph G is defined as the new graph obtained by adding an extra vertex into each edge of G .

Definition 1.3. [18] For a graph G with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$, we take another copy of G with vertices labelled by $\{v_1, v_2, \dots, v_n\}$, this time, where v_i corresponds to v_i for each i . If we connect v_i to the neighbours of v_i for each i , we obtain a new graph called the double graph of G and it is denoted by $D(G)$.

In this paper we study the Reduced Sombor index for different graph structures. We also find this molecular descriptor for Graphene and Carbon nanocones.

In Graphene each and every atom is available for chemical reaction from two sides. It is of the honeycomb lattice structure formed by the carbon atoms. Due to it's property of electron mobility, it can be used in electronic devices. Here we obtained the Reduced Sombor index for the structure Graphene.

Carbon nanocones are very important structures whose bases are attached to the graphite. Those are of the laboratory made nanocones. In gas sensors, biosensors and to store the energy, these nanostructures will be used. Throughout the paper we use $P_n, S_n^0, K_{n \times 2}, C_n, K_n, K_{1,n-1}, F_n^3, D_n^3$ and $T_{(p,q)}$ to represent Path, Crown graph, Cocktail party graph, Cycle, Complete graph, Star graph, Friendship graph, Dutch windmill graph, and Tadpole graph respectively.

II. REDUCED SOMBOR INDEX OF SOME STANDARD GRAPHS

Theorem 2.1.

$$RS(G) = \begin{cases} (n-3)\sqrt{2} + 2 & \text{if } G = P_n \\ n(n-1)(n-2)\sqrt{2} & \text{if } G = S_n^0 \\ 2n(n-1)(2n-3)\sqrt{2} & \text{if } G = K_{n \times 2} \\ n\sqrt{2} & \text{if } G = C_n \\ (n-2)6\sqrt{2} & \text{if } G = K_n \\ n^2 - 3n + 2 & \text{if } G = K_{1,n-1} \\ 2n \left[\sqrt{4n^2 - 4n - 2} \right] + n\sqrt{2} & \text{if } G = F_n^3 \\ 2n \left[\sqrt{4n^2 - 4n - 2} \right] + 2n\sqrt{2} & \text{if } G = D_n^3 \\ 3\sqrt{5} + (p+q-4)\sqrt{2} + 1 & \text{if } G = T_{(p,q)} \end{cases}$$

Proof. We prove the theorem for Cocktail party graphs. Same proof methods can be used for others. In Cocktail party graphs, $2n(n-1)$ edges are there between the vertices of degree $2n-2$. Thus by the definition

$$RS = 2n(n-1)(2n-3)\sqrt{2}$$

□

III. REDUCED SOMBOR INDEX OF DOUBLE GRAPHS

Theorem 3.1.

$$RS(D(G)) = \begin{cases} 8\sqrt{10} + (4n-12)8\sqrt{2} & \text{if } G = P_n, n \geq 3 \\ 12\sqrt{2}n & \text{if } G = C_n \\ 2n(n-1)(2n-3)\sqrt{2} & \text{if } G = K_n \end{cases}$$

Proof. We prove the theorem for Double graph of Cycle. Let G be a Double graph of Cycle $D(C_n)$. $4n$ edges are there in between the vertices of degree 4. By the definition of Reduced Sombor index,

$$RS(D(C_n)) = 12\sqrt{2}n$$

IV. REDUCED SOMBOR INDEX OF COMPLEMENTS

Here we use the 1-complements of Star graph, Cocktail party graph and complete graph. The following theorem follows directly from definitions.

Theorem 4.1

$$RS[G] = \begin{cases} \frac{1}{2}(n-1)(n-2)(n-3)\sqrt{2} & \text{if } G = \overline{K_{1,n-1}} \\ 0 & \text{if } G = \overline{K_{n \times 2}}, \\ 0 & \text{if } G = \overline{K_n} \end{cases}$$

V. REDUCED SOMBOR INDEX OF SUBDIVISION GRAPHS

Theorem 5.1

$$RS[(S(G))] = \begin{cases} (n-2)2\sqrt{2} + 2 & \text{if } G = P_n \\ 2n\sqrt{2} & \text{if } G = C_n \\ n(n-1)\sqrt{n^2 - 4n + 5} & \text{if } G = K_n \\ (n-1) \left[1 + \sqrt{n^2 - 4n + 3} \right] & \text{if } G = K_{1, n-1} \end{cases}$$

Proof. We prove the theorem for subdivision graph of Cycle graph. Let G be a subdivision graph of Cycle graph $S(C_n)$. Here all the vertices are with degree 2. Thus by the definition of Reduced Sombor index,

$$RS(S(C_n)) = 2n\sqrt{2}$$

□

VI. REDUCED SOMBOR INDEX OF GRAPHENE AND CARBON NANOCONES

Theorem 6.1. Reduced Sombor index of Graphene with 'a' rows of benzene rings and 'b' benzene rings in each row is given by

$$RS(\text{Graphene}) = \begin{cases} (a+4)\sqrt{2} + (2a+4b-4)\sqrt{5} + (3ab-2b-a-1)2\sqrt{2} & \text{if } a \neq 1 \\ 4\sqrt{2} + (4b-4)\sqrt{5} + 2\sqrt{2}b & \text{if } a = 1 \end{cases}$$

Proof. Graphene is having 'a' rows and 'b' benzene rings in each row. It has $(a+4)$ edges between the vertices of degree 2 each, $2a+4b-4$ edges between the vertices with degree 2 and 3. Graphene also has $3ab-2b-a-1$ edges in between the vertices with common degree 3. Hence, by the definition of index one can get

$$RS(\text{Graphene}) = \begin{cases} (a+4)\sqrt{2} + (2a+4b-4)\sqrt{5} + (3ab-2b-a-1)2\sqrt{2} & \text{if } a \neq 1 \\ 4\sqrt{2} + (4b-4)\sqrt{5} + 2\sqrt{2}b & \text{if } a = 1 \end{cases}$$

□

The proof of the following theorem is similar to the proof of Theorem 6.1

Theorem 6.2 For $[CNC_k[n]]$ nanocones with $k \geq 3$ and $n = 1, 2, 3, \dots$,

$$RS[CNC_k[n]] = k + 2k\sqrt{5} + \sqrt{2}k(3n^2 - 5n + 2)$$

VII. STARPHENE GRAPH

Starphene graph $St(p, q, r)$ where p, q, r represents the total number of hexagons in every linear chain joined by a single together by central hexagon. More details on topological indices of Starphene graph can be seen in [2]. The molecular structure of Starphene is represented below.

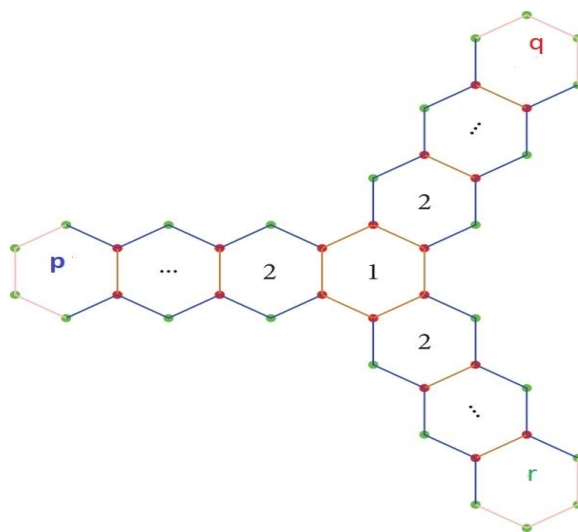


Fig 1. Starphene graph $St(p, q, r)$

Theorem 7.1. For Starphene graph $St(p, q, r)$ with three components p, q and r ,

$$St(p, q, r) = 9\sqrt{2} + (4\sqrt{5} + 2\sqrt{2})(p + q + r) - 9\sqrt{5}$$

Proof. The Starphene graph $St(p, q, r)$ is having nine edges between the vertices with degree 2. There are $2(2p + 2q + 2r - 9)$ edges between the vertices with degree 2 and 3. $(p + q + r)$ edges are present between the vertices with degree 3 each. Thus the proof follows by the definition.

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