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LE-Degree Product Energy of Graph

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***Abstract*— In this paper we introduce and investigate the LE-Degree Product energy of a graph $\mathbf{LEDPE(G)}$. We establish upper and lower bounds for $\mathbf{LEDPE(G)}$.**

***Index Terms*— LE-Degree Product index, LE-Degree Product eigenvalues, LE-Degree Product energy, k-complement.**

I. INTRODUCTION

Matrix representation of a graph set a new trend in the research of Mathematical Chemistry. Adjacency matrix is also known as $0-1$ matrix whose entries will be 1, if the two corresponding vertices are adjacent and for other case the entry will be 0. Energy of graph is the sum of absolute values of eigenvalues extracted by the adjacency matrix of graph. Thousands of studies have been published since the beginning of graph energy (See for example [1-4, 6-9, 23]).

In the recent decades, Topological indices attracted the researchers of both mathematics as well as Chemistry due to their numerous applications. For new topological indices, we suggest the reader to refer the papers [10-21]. LE-Degree Product is a new molecular descriptor, introduced by K. N. Prakasha [5]. He defined the new index of a graph G as follows

$$LEDPM(G) = LEDP(G) = \sum_{i\sim j} (\ln(d_u d_v) + e^{d_u d_v}).$$

Here d_u and d_v represent the degree of vertex u and v respectively. By the motivation of the LE-Degree Product index, we associate a symmetric square matrix $LEDP(G)$ to a graph G. The LE-Degree Product matrix $LEDP(G) = \left(I_{ij}\right)_{n\times n}$ is defined as

$$= \begin{Bmatrix} \ln(d_u d_v) + e^{d_u d_v} & uv \in E \\ 0 & otherwise \end{Bmatrix}$$

II. THE LE-DEGREE PRODUCT ENERGY OF A GRAPH

Throughout the paper we considered a graph G which is a simple, finite, undirected, sign-less, mark-less graph.

Let $LEDP(G)$ be the LE-Degree Product matrix. The characteristic polynomial of $LEDP(G)$ will be denoted by $\phi_{LEDP}(G,\lambda)$ and defined as

$$\phi_{LEDP}(G,\lambda) = det\left(\lambda I - LEDP(G)\right).$$

Since the LE-Degree Product matrix is real and symmetric, its eigenvalues are real numbers, and we label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. The LE-Degree Product energy of G is similarly defined by

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$$LEDPE(G) = \sum_{i=1}^{n} |\lambda_i|.$$

This paper is organized as follows. In Section 3, we give the basic properties of LE-Degree Product energy of a graph. In Section 4, LE-Degree Product energy of some standard and well-studied graphs are obtained. In Section 5, we compute inverse sum connectivity energy of some complements of some specific graphs.

Some basic properties of LE-Degree Product energy of a graph

Proposition 1. *The first three coefficients of the polynomial $\phi_{LEDP}(G,\lambda)$ are as follows:*
(i) $a_0 = 1$,
(ii) $a_1 = 0$,
(iii) $a_2 = -\sum_{i<j}[\ln(d_u d_v) + e^{d_u d_v}]^2$.

Proof. (i) By the definition of $\Phi_{LEDP}(G,\lambda) = det[\lambda I - LEDP(G)]$, we get $a_0 = 1$.
(ii) The sum of determinants of all 1×1 principal submatrices of $LEDP(G)$ is equal to the trace of $LEDP(G)$ implying that

$$a_1 = (-1)^1 \times \text{the trace of } LEDP(G) = 0.$$

(iii) By the definition, we have

$$(-1)^2 a_2 = \sum_{1\le i<j\le n} \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix}$$

$$a_2 = \sum_{1\le i<j\le n} a_{ii}\, a_{jj} - a_{ji} a_{ij}$$

$$a_2 = \sum_{1\le i<j\le n} a_{ii}\, a_{jj} - \sum_{1\le i<j\le n} a_{ji}\, a_{ij}$$

$$a_2 = -\sum_{i<j}[\ln(d_u d_v) + e^{d_u d_v}]^2.$$

□

Proposition 2. *If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the inverse sum connectivity eigenvalues of $LEDP(G)$, then $\sum_{i=1}^{n} {\lambda_i}^2 = 2\sum_{i<j}[\ln(d_u d_v) + e^{d_u d_v}]^2$.*

Proof. It follows as

$$\sum_{i=1}^{n} \lambda_i^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\, a_{ji}$$

$$\sum_{i=1}^{n} \lambda_i^2 = 2\sum_{i<j} \left(a_{ij}\right)^2 + \sum_{i=1}^{n} (a_{ii})^2$$

$$\sum_{i=1}^{n} \lambda_i^2 = 2\sum_{i<j} \left(a_{ij}\right)^2$$

$$\sum_{i=1}^{n} \lambda_i^2 = 2\sum_{i<j}[\ln(d_u d_v) + e^{d_u d_v}]^2.$$

□

Using this result, we now obtain lower and upper bounds for the LE-Degree Product energy of a graph:

Theorem 3. *Let G be a graph with n vertices. Then $LEDP(G) \le \sqrt{2n\sum_{i<j}[\ln(d_u d_v) + e^{d_u d_v}]^2}$.*

Theorem 4. *Let G be a graph with n vertices, then $LEDPE(G) \ge \sqrt{2\sum_{i<j}[\ln(d_u d_v) + e^{d_u d_v}]^2 + n(n-1)\left[Det\left(LEDP(G)\right)\right]^{\frac{2}{n}}}$.*

III. LE-Degree Product Energy of Some Standard Graphs

Theorem 5. *The LE-Degree Product energy of a complete graph K_n is $LEDPE(K_n) = 2(n-1)\left(\ln((n-1)^2) + e^{(n-1)^2}\right)$.*

Proof. Let K_n be the complete graph with vertex set $V = \{v_1, v_2, \cdots, v_n\}$. For this graph, the LE-Degree Product matrix is

$$\begin{bmatrix} 0 & A & A & \dots & A & A \\ A & 0 & A & \dots & A & A \\ A & A & 0 & \dots & A & A \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ A & A & A & \dots & 0 & A \\ A & A & A & \dots & A & 0 \end{bmatrix}.$$

The characteristic equation then becomes

$$\left(\lambda + \ln((n-1)^2) + e^{(n-1)^2}\right)^{n-1}\left(\lambda - (n-1)\ln((n-1)^2) + e^{(n-1)^2}\right) = 0$$

and the spectrum would be

$$Spec_{LEDP}(K_n) = \begin{pmatrix} -\ln((n-1)^2) + e^{(n-1)^2} & (n-1)\ln((n-1)^2) + e^{(n-1)^2} \\ n-1 & 1 \end{pmatrix}.$$

Therefore, $LEDPE(K_n) = 2(n-1)\left(\ln((n-1)^2) + e^{(n-1)^2}\right)$. □

Theorem 6. *The LE-Degree Product energy of the star graph* $K_{1,n-1}$ *is* $LEDPE(K_{1,n-1}) = 2[\ln(n-1) + e^{n-1}]\sqrt{(n-1)}$.

Proof. Let $K_{1,n-1}$ be the star graph with vertex set $V = \{v_0, v_1, \cdots, v_{n-1}\}$ with v_0 denotes the central vertex. The LE-Degree Product matrix is

$$\begin{bmatrix} 0 & \ln(n-1) + e^{n-1} & \ln(n-1) + e^{n-1} & \dots & \ln(n-1) + e^{n-1} & \ln(n-1) + e^{n-1} \\ \ln(n-1) + e^{n-1} & 0 & 0 & \dots & 0 & 0 \\ \ln(n-1) + e^{n-1} & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \ln(n-1) + e^{n-1} & 0 & 0 & \dots & 0 & 0 \\ \ln(n-1) + e^{n-1} & 0 & 0 & \dots & 0 & 0 \end{bmatrix}.$$

The characteristic equation becomes

$$\lambda^{n-2}(\lambda^2 - (n-1)[\ln(n-1) + e^{n-1}]^2) = 0$$

and therefore, the spectrum would be

$$Spec_{LEDP}(K_{1,n-1}) = \begin{pmatrix} -[\ln(n-1) + e^{n-1}]\sqrt{(n-1)} & 0 & [\ln(n-1) + e^{n-1}]\sqrt{(n-1)} \\ 1 & n-2 & 1 \end{pmatrix}.$$

Therefore,

$$LEDPE(K_{1,n-1}) = 2[\ln(n-1) + e^{n-1}]\sqrt{(n-1)}.$$

□

Theorem 7. *The LE-Degree Product energy of crown graph* S_n^0 *is* $LEDPE(S_n^0) = (n-1)\left(\ln((n-1)^2) + e^{(n-1)^2}\right)$.

Proof. Let S_n^0 be the crown graph of order $2n$ and let the vertex set of this graph be $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$. The LE-Degree Product matrix of S_n^0 is

$$\begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & B & \dots & B & B \\ 0 & 0 & 0 & \dots & 0 & B & 0 & \dots & B & B \\ 0 & 0 & 0 & \dots & 0 & B & B & \dots & 0 & B \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & B & B & \dots & B & 0 \\ 0 & B & B & \dots & B & 0 & 0 & \dots & 0 & 0 \\ B & 0 & B & \dots & B & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ B & B & 0 & \dots & B & 0 & 0 & \dots & 0 & 0 \\ B & B & B & \dots & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}.$$

Here $B = \ln((n-1)^2) + e^{(n-1)^2}$

Therefore, the characteristic equation is

$$(\lambda - B)^{n-1}(\lambda + B)^{n-1}\left(\lambda + (n-1)(B)\right)\left(\lambda - (n-1)(B)\right) = 0$$

implying that the spectrum is

$$Spec_{LEDP}(S_n^0) = \begin{pmatrix} (n-1)(B) & -(n-1)(B) & B & -B \\ 1 & 1 & n-1 & n-1 \end{pmatrix}.$$

Therefore, we obtain

$$LEDPE(S_n^0) = 2(n-1)\left(\ln((n-1)^2) + e^{(n-1)^2}\right).$$

□

Theorem 8. *The LE-Degree Product energy of the cocktail party graph* $K_{n\times 2}$ *is* $RSE(K_{n\times 2}) = 4(n-1)\left[\ln(4(n-1)^2) + e^{4(n-1)^2}\right]$.

Proof. Let $K_{n\times 2}$ be the cocktail party graph of order $2n$ with vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$.
The LE-Degree Product matrix is

$$\begin{bmatrix} 0 & C & C & C & \dots & 0 & C & C & C \\ C & 0 & C & C & \dots & C & 0 & C & C \\ C & C & 0 & C & \dots & C & C & 0 & C \\ C & C & C & 0 & \dots & C & C & C & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & C & C & C & \dots & 0 & C & C & C \\ C & 0 & C & C & \dots & C & 0 & C & C \\ C & C & 0 & C & \dots & C & C & 0 & C \\ C & C & C & 0 & \dots & C & C & C & 0 \end{bmatrix}.$$

Where $C = \ln(4(n-1)^2) + e^{4(n-1)^2}$.

This implies that the characteristic equation becomes

$$\lambda^n(\lambda + 2[C])^{n-1}(\lambda - 2(n-1)[C]) = 0.$$

Hence, the spectrum is

$$Spec_{RS}(K_{n\times 2}) = \begin{pmatrix} -2[C] & 0 & 2(n-1)[C] \\ n-1 & n & 1 \end{pmatrix}.$$

Therefore,

$$RSE(K_{n\times 2}) = 4(n-1)\left[\ln(4(n-1)^2) + e^{4(n-1)^2}\right].$$

□

Theorem 9. *The LE-Degree Product energy of the complete bipartite graph* $K_{m,n}$ *of order* $m \times n$ *with vertex set* $\{u_1, u_2, \cdots, u_m, v_1, v_2, \cdots, v_n\}$ *is* $LEDPE\left(K_{m,n}\right) = 2\sqrt{mn}[\ln mn + e^{mn}]$.

Proof. The LE-Degree Product matrix of the complete bipartite graph $K_{m,n}$ is

$$\begin{bmatrix} 0 & 0 & 0 & \dots & \ln mn + e^{mn} & \ln mn + e^{mn} & \ln mn + e^{mn} \\ 0 & 0 & 0 & \dots & \ln mn + e^{mn} & \ln mn + e^{mn} & \ln mn + e^{mn} \\ 0 & 0 & 0 & \dots & \ln mn + e^{mn} & \ln mn + e^{mn} & \ln mn + e^{mn} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \ln mn + e^{mn} & \ln mn + e^{mn} & \ln mn + e^{mn} & \dots & 0 & 0 & 0 \\ \ln mn + e^{mn} & \ln mn + e^{mn} & \ln mn + e^{mn} & \dots & 0 & 0 & 0 \\ \ln mn + e^{mn} & \ln mn + e^{mn} & \ln mn + e^{mn} & \dots & 0 & 0 & 0 \end{bmatrix}.$$

Then the characteristic equation is

$$\lambda^{m+n-2}\left(\lambda - \sqrt{mn}[\ln mn + e^{mn}]\right)\left(\lambda + \sqrt{mn}[\ln mn + e^{mn}]\right) = 0$$

and therefore the spectrum is

$$Spec_{LEDP}\left(K_{m,n}\right) = \begin{pmatrix} \sqrt{mn}[\ln mn + e^{mn}] & 0 & -\sqrt{mn}[\ln mn + e^{mn}] \\ 1 & m+n-2 & 1 \end{pmatrix}.$$

Therefore, we get

$$LEDPE\left(K_{m,n}\right) = 2\sqrt{mn}[\ln mn + e^{mn}].$$

□

IV. LE-DEGREE PRODUCT ENERGY OF COMPLEMENTS

Definition 10. Let G be a graph and $\mathrm{P_k} = \{\mathrm{V_1}, \mathrm{V_2}, \cdots, \mathrm{V_k}\}$ be a partition of its vertex set V. Then the k-complement of G is denoted by $\overline{(\mathrm{G})_{\mathrm{k}}}$ and obtained as follows: For all $\mathrm{V_i}$ and $\mathrm{V_j}$ in $\mathrm{P_k}$, $\mathrm{i} \neq \mathrm{j}$, remove the edges between $\mathrm{V_i}$ and $\mathrm{V_j}$ and add the edges between the vertices of $\mathrm{V_i}$ and $\mathrm{V_j}$ which are not in G.

Definition 11. Let G be a graph and $\mathrm{P_k} = \{\mathrm{V_1}, \mathrm{V_2}, \cdots, \mathrm{V_k}\}$ be a partition of its vertex set V. Then the $\mathrm{k(i)}$-complement of G is denoted by $\overline{(\mathrm{G})_{\mathrm{k(i)}}}$ and obtained as follows: For each set $\mathrm{V_r}$ in $\mathrm{P_k}$, remove the edges of G joining the vertices within $\mathrm{V_r}$ and add the edges of $\overline{\mathrm{G}}$ (complement of G) joining the vertices of $\mathrm{V_r}$.

Theorem 12. The LE-Degree Product energy of the complement $\overline{K_n}$ of the complete graph K_n is $LEDPE(\overline{K_n}) = 0$.

Proof. Let K_n be the complete graph with vertex set $V = \{v_1, v_2, \cdots, v_n\}$. The LE-Degree Product connectivity matrix of the complement of the complete graph K_n is

$$LEDP(\overline{K_n}) = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}.$$

Clearly, the characteristic equation is $\lambda^n = 0$ implying

$$E(\overline{K_n}) = 0.$$

□

Theorem 13. *The LE-Degree Product energy of the complement* $\overline{K_{1,n-1}}$ *of the star graph* $K_{1,n-1}$ *is* $LEDPE(\overline{K_{1,n-1}}) = 2(n-2)[\ln(n-2)^2 + e^{(n-2)^2}]$.

Proof. Let $\overline{K_{1,n-1}}$ be the complement of the star graph with vertex set $V = \{v_0, v_1, \cdots, v_{n-1}\}$ where v_0 is the central. The LE-Degree Product matrix is

$$\begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \ln(n-2)^2 + e^{(n-2)^2} & \dots & \ln(n-2)^2 + e^{(n-2)^2} & \ln(n-2)^2 + e^{(n-2)^2} \\ 0 & \ln(n-2)^2 + e^{(n-2)^2} & 0 & \dots & \ln(n-2)^2 + e^{(n-2)^2} & \ln(n-2)^2 + e^{(n-2)^2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \ln(n-2)^2 + e^{(n-2)^2} & \ln(n-2)^2 + e^{(n-2)^2} & \dots & 0 & \ln(n-2)^2 + e^{(n-2)^2} \\ 0 & \ln(n-2)^2 + e^{(n-2)^2} & \ln(n-2)^2 + e^{(n-2)^2} & \dots & \ln(n-2)^2 + e^{(n-2)^2} & 0 \end{bmatrix}.$$

The corresponding characteristic equation is

$$\lambda\left(\lambda - (n-2)[\ln(n-2)^2 + e^{(n-2)^2}]\right)\left(\lambda + \ln(n-2)^2 + e^{(n-2)^2}\right)^{n-2} = 0$$

and therefore the spectrum is

$$Spec_{LEDP}(\overline{K_{1,n-1}}) = \begin{pmatrix} -\ln(n-2)^2 + e^{(n-2)^2} & 0 & (n-2)[\ln(n-2)^2 + e^{(n-2)^2}] \\ n-2 & 1 & 1 \end{pmatrix}.$$

Therefore,

$$LEDPE(\overline{K_{1,n-1}}) = 2(n-2)[\ln(n-2)^2 + e^{(n-2)^2}].$$

□

Theorem 14. *The LE-Degree Product energy of the complement* $\overline{K_{n\times 2}}$ *of the cocktail party graph* $K_{n\times 2}$ *of order* $2n$ *is* $LEDPE(\overline{K_{n\times 2}}) = 2ne$.

Proof. Let $K_{n\times 2}$ be the cocktail party graph of order $2n$ having the vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$. The corresponding LE-Degree Product matrix is

$$LEDP(\overline{K_{n\times 2}}) = \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & e & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & e \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ e & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & e & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & e & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e & \dots & 0 & 0 & 0 & 0 \end{bmatrix}$$

and the characteristic equation becomes

$$(\lambda + e)^n(\lambda - e)^n = 0$$

implying that the spectrum would be

$$Spec_{LEDP}(\overline{K_{n\times 2}}) = \begin{pmatrix} e & -e \\ n & n \end{pmatrix}.$$

Therefore,

$$LEDPE(\overline{K_{n\times 2}}) = 2ne.$$

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