

Connectivity Index of Eugenol

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Abstract—Chemists can use graph theory to obtain different connectivity indices. A Connectivity index $Con(G_1)$ of a graph G_1 is the property that states every graph G_1 is isomorphic to G_2 , $Con(G_1) = Con(G_2)$. In this article, we calculate various indices for Eugenol, such as ABC index, ABC4 index, Sum connectivity index, GA index, Randic index, GA5 index, 1st and 2nd Zagreb index, 1st and 2nd Multiple Zagreb index, Augmented Zagreb index, Harmonic index, Hyper Zagreb index, 1st, 2nd and 3rd Zagreb polynomials, Forgotten connectivity index and polynomials and Symmetric division index.

Index Terms— Connectivity indices.

I. INTRODUCTION

Clove oil, or Eugenol, is a fragrant liquid that comes from cloves. It has many uses, such as flavouring foods and teas, treating toothache, and easing gastrointestinal and respiratory problems. Its molecular formula is $C_{10}H_{12}O_2$, as shown in Fig.1.

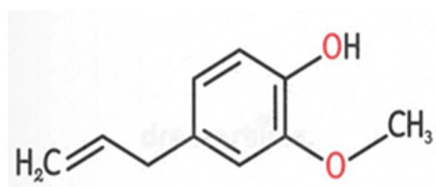


Fig. 1

We can obtain topological forms by removing hydrogen atoms from molecular graphs, where vertices represent atoms and edges represent bonds. Different types of topological matrices can describe the relationships between the atoms. We can manipulate these matrices mathematically to get a single number, called a graph invariant or a topological index. In this paper, we only consider molecular graphs that are finite, connected, and have no loops. Let $G_1 = (V, E)$ be a graph with a set of vertices and edges. The degree of a vertex $u \in V(G_1)$, The notation uv denotes an edge connects the vertices u and v .

II. LITERATURE REVIEW

The ABC index is a degree-based molecular form that Estrada^[7] proposed in 1990. It can model the thermodynamic properties of organic chemical compounds^[20] and explain the stability of branched alkanes^[8]. Some upper bounds for this index of graphs are in^[3]. The geometric-arithmetic (GA) index is a new graph invariant in mathematical chemistry^[2]. The ABC index of bicyclic and connected graphs is in^[4,30]. More results on this index of trees are in^[11-37]. M. Ghorbani et., al^[15] introduced the fourth ABC index in 2010. More studies on this index are in^[9,10]. M Randic^[23] introduced the degree-based topological index, which measures the branching of molecules. The sum connectivity index is a Randic-like index that Zhou and N. Trinajstić^[33] introduced. More studies on this index are in^[34,35]. D. Vukicević et.al^[27] introduced the GA index of a graph G. More studies on this index are in^[25,32]. A. Graovac et., al^[16] introduced the fifth GA index in 2011. The 1st and 2nd Zagreb index are molecular descriptors that I. Gutman and N. Trinajstić^[17] introduced in 1972 in the connectivity formula for the total π -energy of conjugated molecules. These indices are useful for QSPR and QSAR studies. Programs like POLLY, DRAGON, CERIUS, TAM, and DISSI can compute them. Z1(G1) and Z2(G1) measure the carbon branching atom molecular skeleton^[20] and can model structure-property relationships. More applications of these indices are in^[25,26]. More studies on these indices are in^[1,18,33,34,35]. Various Zagreb2topological indices of graph G1 are based on the degree of its vertices and are in^[14]. The 1st and 2nd second Several Zagreb indices properties are in^[6,19]. Furtula et.,al^[12] introduced the Augmented Zagreb index. Researchers have developed a new connectivity index in chemical graph theory that has shown to be more effective in predicting the heat of formation in octanes and heptanes than the atom-bond connectivity index. This new index exhibits superior correlation properties and structural sensitivity compared to other well-established degree-based topological indices. These findings suggest that the new index has the potential to make significant contributions to the field of chemical graph theory.

The correlation between the Harmonic index, Randic index, and π -electron energy of benzenoid hydrocarbons was investigated in the study^[36]. Other studies on the Harmonic index include^[28,34]. In a study by Shirdel et al.^[24], a new distance-based Zagreb index called the Hyper-Zagreb Index was defined for graph G1. Fath-Tabar^[37] introduced the Third Zagreb index and the 1st, 2nd, and 3rd Zagreb Polynomials in 2011. Another degree-based topological index for simple graphs is called the forgotten topological index introduced in^[13].

III. RESULTS

In this article, we find some connectivity indices of $C_{10}H_{12}O_2$

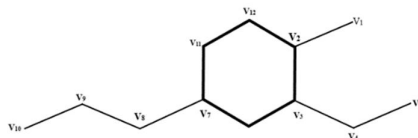


Fig. 2

Theorem. 1. The connectivity index of Eugenol for Atom bond is 8.554231.

Proof: Consider Eugenol ($C_{10}H_{12}O_2$). The two-dimensional structure of Eugenol (Fig. 2) has 5 edges of the types $n_{1,2}$, $n_{1,3}$, $n_{2,2}$, $n_{2,3}$ and $n_{3,3}$, where n_{ij} denotes the edges that connect vertices with degrees d_i and d_j . From the Fig. 2, the following are the numbers of these types of edges: $|n_{1,2}| = 2$; $|n_{1,3}| = 1$; $|n_{2,2}| = 2$; $|n_{2,3}| = 6$; $|n_{3,3}| =$

1. The atom-bond connectivity index of Eugenol is $|n_{i,j}| \sum_{uv \in E(G_1)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$

$$\begin{aligned}
 &= |n_{1,2}| \left(\sqrt{\frac{1+2-2}{1 \cdot 2}} \right) + |n_{1,3}| \left(\sqrt{\frac{1+3-2}{1 \cdot 3}} \right) + |n_{2,2}| \left(\sqrt{\frac{2+2-2}{2 \cdot 2}} \right) + |n_{2,3}| \left(\sqrt{\frac{2+3-2}{2 \cdot 3}} \right) + |n_{3,3}| \left(\sqrt{\frac{3+3-2}{3 \cdot 3}} \right) \\
 &= 2 \sqrt{\frac{1}{2}} + 1 \sqrt{\frac{2}{3}} + 2 \sqrt{\frac{1}{2}} + 6 \sqrt{\frac{1}{2}} + 1 \sqrt{\frac{4}{9}} = 8.554231 \\
 &\therefore ABC(C_{10}H_{12}O_2) = 8.554231
 \end{aligned}$$

Theorem. 2. The connectivity index of Eugenol for 4th atom bond is. 6.996657.

Proof: The edges of Eugenol is denoted by e_{ij} with $i = S_u$ and $j = S_v$. The sum of the degrees of the edge end points of Eugenol has nine different types of edges. $e_{2,3}$, $e_{2,4}$, $e_{3,5}$, $e_{3,6}$, $e_{4,7}$, $e_{5,5}$, $e_{5,6}$, $e_{6,6}$, $e_{6,7}$ as shown in the following Fig. 2. from the Fig. 2., $|e_{2,3}| = 1$; $|e_{2,4}| = 1$; $|e_{3,5}| = 1$; $|e_{3,6}| = 1$; $|e_{4,7}| = 1$; $|e_{5,5}| = 1$; $|e_{5,6}| = 3$; $|e_{6,6}| = 1$; $|e_{6,7}| = 2$ The 4th atom-bond connectivity index of Eugenol = $ABC_4(G_1) = |e_{i,j}| \sum_{uv \in E(G_1)} \sqrt{\frac{s_u + s_v - 2}{s_u s_v}}$

$$\begin{aligned}
 &= |e_{2,3}| \left(\sqrt{\frac{2+3-2}{2 \cdot 3}} \right) + |e_{2,4}| \left(\sqrt{\frac{2+4-2}{2 \cdot 4}} \right) + |e_{3,5}| \left(\sqrt{\frac{3+5-2}{3 \cdot 5}} \right) + |e_{3,6}| \left(\sqrt{\frac{3+6-2}{3 \cdot 6}} \right) \\
 &+ |e_{4,7}| \left(\sqrt{\frac{4+7-2}{4 \cdot 7}} \right) + |e_{5,5}| \left(\sqrt{\frac{5+5-2}{5 \cdot 5}} \right) + |e_{5,6}| \left(\sqrt{\frac{5+6-2}{5 \cdot 6}} \right) + |e_{6,6}| \left(\sqrt{\frac{6+6-2}{6 \cdot 6}} \right) + |e_{6,7}| \left(\sqrt{\frac{6+7-2}{6 \cdot 7}} \right) \\
 &= 1 \left(\sqrt{\frac{1}{2}} \right) + 1 \left(\sqrt{\frac{1}{2}} \right) + 1 \left(\sqrt{\frac{2}{5}} \right) + 1 \left(\sqrt{\frac{7}{18}} \right) + 1 \left(\sqrt{\frac{9}{28}} \right) + 1 \left(\sqrt{\frac{8}{25}} \right) + 3 \left(\sqrt{\frac{9}{30}} \right) + 1 \left(\sqrt{\frac{10}{36}} \right) + 2 \left(\sqrt{\frac{11}{42}} \right) = 6.996657 \\
 &\therefore ABC_4(C_{10}H_{12}O_2) = 6.996657
 \end{aligned}$$

Theorem. 3. The Eugenol's Randic connectivity index is 5.774387.

Proof: Consider the Eugenol's Randic connectivity index = $\chi = |n_{i,j}| \sum_{uv \in E(G_1)} \frac{1}{\sqrt{d_u d_v}}$

$$\begin{aligned}
 &= |n_{1,2}| \left(\sqrt{\frac{1}{1 \cdot 2}} \right) + |n_{1,3}| \left(\sqrt{\frac{1}{1 \cdot 3}} \right) + |n_{2,2}| \left(\sqrt{\frac{1}{2 \cdot 2}} \right) + |n_{2,3}| \left(\sqrt{\frac{1}{2 \cdot 3}} \right) + |n_{3,3}| \left(\sqrt{\frac{1}{3 \cdot 3}} \right) \\
 &= 2 \left(\sqrt{\frac{1}{2}} \right) + 1 \left(\sqrt{\frac{1}{3}} \right) + 2 \left(\sqrt{\frac{1}{4}} \right) + 6 \left(\sqrt{\frac{1}{6}} \right) + 1 \left(\sqrt{\frac{1}{9}} \right) \\
 &= \sqrt{2} + \sqrt{\frac{1}{3}} + 1 + \sqrt{6} + \sqrt{\frac{1}{9}} = 5.774387 \\
 &\therefore \chi(C_{10}H_{12}O_2) = 5.774387
 \end{aligned}$$

Theorem. 4. The Eugenol's sum connectivity index is 5.74623.

Proof: Consider the Eugenol's sum connectivity index = $S(C_{10}H_{12}O_2)$

$$\begin{aligned}
 &= |m_{i,j}| \left(\sum_{uv \in E(G_1)} \frac{1}{\sqrt{d_u + d_v}} \right) \\
 &= |n_{1,2}| \left(\frac{1}{\sqrt{1+2}} \right) + |n_{1,3}| \left(\frac{1}{\sqrt{1+3}} \right) + |n_{2,2}| \left(\frac{1}{\sqrt{2+2}} \right) + |n_{2,3}| \left(\frac{1}{\sqrt{2+3}} \right) + |n_{3,3}| \left(\frac{1}{\sqrt{3+3}} \right) \\
 &= \frac{2}{\sqrt{3}} + \frac{1}{2} + 1 + \frac{6}{\sqrt{5}} + \frac{1}{\sqrt{6}} = 5.746230 \\
 &\therefore S(C_{10}H_{12}O_2) = 5.746230
 \end{aligned}$$

Theorem. 5. The Eugenol's Geometric-Arithmetic index is 13.630419.

Proof: Consider the Eugenol's Geometric-Arithmetic index = $G = |n_{i,j}| \sum_{uv \in E(G_1)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$

$$\begin{aligned}
 &= |n_{1,2}| \left(\frac{2\sqrt{1 \cdot 2}}{1+2} \right) + |n_{1,3}| \left(\frac{2\sqrt{1 \cdot 3}}{1+3} \right) + |n_{2,2}| \left(\frac{2\sqrt{2 \cdot 2}}{2+2} \right) + |n_{2,3}| \left(\frac{2\sqrt{2 \cdot 3}}{2+3} \right) + |n_{3,3}| \left(\frac{2\sqrt{3 \cdot 3}}{3+3} \right) \\
 &= 2 \left(\frac{2\sqrt{2}}{3} \right) + 1 \left(\frac{2\sqrt{3}}{4} \right) + 2 \left(\frac{2\sqrt{4}}{4} \right) + 6 \left(\frac{2\sqrt{6}}{5} \right) + 1 \left(\frac{2\sqrt{9}}{6} \right) \\
 &= \frac{4\sqrt{2}}{3} + \frac{\sqrt{3}}{2} + 2 + \frac{12\sqrt{6}}{5} + 3 = 13.630419 \\
 &\therefore GA(C_{10}H_{12}O_2) = 13.630419
 \end{aligned}$$

Theorem. 6. The Eugenol's 5th Geometric-Arithmetic index is 11.409858.

Proof: Consider the Eugenol's 5th Geometric-Arithmetic index = $GA_5 = |e_{i,j}| \sum_{uv \in E(G_1)} \frac{2\sqrt{s_u s_v}}{s_u + s_v}$

$$\begin{aligned}
 &= |e_{2,3}| \left(\frac{2\sqrt{2 \cdot 3}}{2+3} \right) + |e_{2,4}| \left(\frac{2\sqrt{2 \cdot 4}}{2+4} \right) + |e_{3,5}| \left(\frac{2\sqrt{3 \cdot 5}}{3+5} \right) + |e_{3,6}| \left(\frac{2\sqrt{3 \cdot 6}}{3+6} \right) + |e_{4,7}| \left(\frac{2\sqrt{4 \cdot 7}}{4+7} \right) \\
 &+ |e_{5,5}| \left(\frac{2\sqrt{5 \cdot 5}}{5+5} \right) + |e_{5,6}| \left(\frac{2\sqrt{5 \cdot 6}}{5+6} \right) + |e_{6,6}| \left(\frac{2\sqrt{6 \cdot 6}}{6+6} \right) + |e_{6,7}| \left(\frac{2\sqrt{6 \cdot 7}}{6+7} \right) \\
 &= 1 \left(\frac{\sqrt{24}}{5} \right) + 1 \left(\frac{\sqrt{32}}{6} \right) + 1 \left(\frac{\sqrt{60}}{8} \right) + 1 \left(\frac{\sqrt{72}}{9} \right) + 1 \left(\frac{\sqrt{112}}{11} \right) + 1 \left(\frac{\sqrt{40}}{10} \right) + 3 \left(\frac{\sqrt{120}}{11} \right) + 1 \left(\frac{\sqrt{144}}{12} \right) + 2 \left(\frac{\sqrt{168}}{13} \right) = 11.409858 \\
 &\therefore GA_5(C_{10}H_{12}O_2) = 11.409858
 \end{aligned}$$

Theorem. 7. The Eugenol's 1st Zagreb index is 54.

Proof: Consider the Eugenol's 1st Zagreb index $= Z_1 = \sum_{uv \in E(G_1)} (d_u + d_v)$
 $= |n_{1,2}|(1+2) + |n_{1,3}|(1+3) + |n_{2,2}|(2+2) + |n_{2,3}|(2+3) + |n_{3,3}|(3+3)$
 $= 2(3) + 1(4) + 2(4) + 6(5) + 1(6) = 6 + 4 + 8 + 30 + 6 = 54$
 $\therefore Z_1(C_{10}H_{12}O_2) = 54$

Theorem. 8. The Eugenol's 2nd Zagreb index is 54.

Proof: Consider the Eugenol's 2nd Zagreb index $= Z_2 = \sum_{uv \in E(G_1)} (d_u d_v)$
 $= |n_{1,2}|(1 \cdot 2) + |n_{1,3}|(1 \cdot 3) + |n_{2,2}|(2 \cdot 2) + |n_{2,3}|(2 \cdot 3) + |n_{3,3}|(3 \cdot 3)$
 $= 2(2) + 1(3) + 2(4) + 6(5) + 1(9) = 4 + 3 + 8 + 30 + 9 = 54$
 $\therefore Z_2(C_{10}H_{12}O_2) = 54$

Theorem. 9. The Eugenol's 1st multiple Zagreb index is $= 5.4 \times 10^7$.

Proof: Consider Eugenol's 1st multiple Zagreb index $= PM_1(C_{10}H_{12}O_2)$
 $= \prod_{uv \in E(G_1)} (d_u + d_v)$
 $= \prod_{e=uv \in 1,2} (d_u + d_v) \prod_{e=uv \in 1,3} (d_u + d_v) \prod_{e=uv \in 2,2} (d_u + d_v) \prod_{e=uv \in 2,3} (d_u + d_v) \prod_{e=uv \in 3,3} (d_u + d_v)$
 $= 3^2 + 4^1 + 4^2 + 5^6 + 6^1 = 5.4 \times 10^7$
 $\therefore PM_1(C_{10}H_{12}O_2) = 5.4 \times 10^7$

Theorem. 10. The Eugenol's 2nd multiple Zagreb index is $= 8.062515 \times 10^7$.

Proof: Consider the Eugenol's 2nd multiple Zagreb index $= PM_2(C_{10}H_{12}O_2)$
 $= \prod_{uv \in E(G_1)} (d_u \cdot d_v)$
 $= \prod_{e=uv \in 1,2} (d_u \cdot d_v) \prod_{e=uv \in 1,3} (d_u \cdot d_v) \prod_{e=uv \in 2,2} (d_u \cdot d_v) \prod_{e=uv \in 2,3} (d_u \cdot d_v) \prod_{e=uv \in 3,3} (d_u \cdot d_v)$
 $= 2^2 + 3^1 + 4^2 + 6^6 + 9^1 = 8.062515 \times 10^7$
 $\therefore PM_2(C_{10}H_{12}O_2) = 8.062515 \times 10^7$

Theorem. 11. The Eugenol's Augmented Zagreb index is 94.765625.

Proof: The Eugenol's augmented Zagreb index $= AZ_1(G_1) = \sum_{uv \in E(G_1)} \left[\frac{d_u d_v}{d_u + d_v - 2} \right]^3$
 $= |n_{1,2}| \left[\frac{1 \cdot 2}{1+2-2} \right]^3 + |n_{1,3}| \left[\frac{1 \cdot 3}{1+3-2} \right]^3 + |n_{2,2}| \left[\frac{2 \cdot 2}{2+2-2} \right]^3 + |n_{2,3}| \left[\frac{2 \cdot 3}{2+3-2} \right]^3 + |n_{3,3}| \left[\frac{3 \cdot 3}{3+3-2} \right]^3$
 $= 2[2]^3 + 1[1.5]^3 + 2[2]^3 + 6[2]^3 + 1 \left[\frac{9}{4} \right]^3 = 16 + 3.375 + 16 + 48 + \frac{729}{64} = 94.765625$
 $\therefore AZ_1(G_1)(C_{10}H_{12}O_2) = 94.765625$

Theorem. 12. The Eugenol's harmonic index is 5.566667.

Proof: The Eugenol's harmonic index $= H = \sum_{uv \in E(G_1)} \frac{2}{d_u + d_v}$
 $= |n_{1,2}| \left[\frac{2}{1+2} \right] + |n_{1,3}| \left[\frac{2}{1+3} \right] + |n_{2,2}| \left[\frac{2}{2+2} \right] + |n_{2,3}| \left[\frac{2}{2+3} \right] + |n_{3,3}| \left[\frac{2}{3+3} \right]$
 $= 2 \left[\frac{2}{3} \right] + 1 \left[\frac{1}{2} \right] + 2 \left[\frac{1}{2} \right] + 6 \left[\frac{2}{5} \right] + 1 \left[\frac{1}{3} \right] = 5.566667$
 $\therefore H(C_{10}H_{12}O_2) = 5.566667$

Theorem. 13. The Eugenol's hyper Zagreb index is 252.

Proof: The Eugenol's hyper Zagreb index $= HM = \sum_{uv \in E(G_1)} (d_u + d_v)^2$
 $= |n_{1,2}|(1+2)^2 + |n_{1,3}|(1+3)^2 + |n_{2,2}|(2+2)^2 + |n_{2,3}|(2+3)^2 + |n_{3,3}|(3+3)^2$
 $= 2(3^2) + 1(4^2) + 2(4^2) + 6(5^2) + 1(6^2) = 252$
 $\therefore HM(C_{10}H_{12}O_2) = 5.566667$

Theorem. 14. The Eugenol's 1st Zagreb polynomials is $x^6 + 6x^5 + 3x^4 + 2x^3$.

Proof: Consider Eugenol's 1st Zagreb polynomials $= ZG_1(G_1, x) = \sum_{uv \in E(G_1)} x^{d_u + d_v}$
 $= |n_{1,2}|x^{1+2} + |n_{1,3}|x^{1+3} + |n_{2,2}|x^{2+2} + |n_{2,3}|x^{2+3} + |n_{3,3}|x^{3+3}$

$$= 2x^3 + x^4 + 2x^4 + 6x^5 + x^6 = x^6 + 6x^5 + 3x^4 + 2x^3$$

$$\therefore ZG_1(G_1, x)(C_{10}H_{12}O_2) = x^6 + 6x^5 + 3x^4 + 2x^3$$

Theorem. 15. The Eugenol's 2nd Zagreb polynomials is $x^9 + 6x^6 + 2x^4 + x^3 + 2x^2$.

Proof: Consider Eugenol's 2nd Zagreb polynomials $= ZG_2(G_2, x) = \sum_{uv \in E(G_1)} x^{d_u \cdot d_v}$

$$= |n_{1,2}|x^{1 \cdot 2} + |n_{1,3}|x^{1 \cdot 3} + |n_{2,2}|x^{2 \cdot 2} + |n_{2,3}|x^{2 \cdot 3} + |n_{3,3}|x^{3 \cdot 3}$$

$$= 2x^2 + x^3 + 2x^4 + 6x^6 + x^9 = x^9 + 6x^6 + 2x^4 + x^3 + 2x^2$$

$$\therefore ZG_2(G_2, x)(C_{10}H_{12}O_2) = x^9 + 6x^6 + 2x^4 + x^3 + 2x^2$$

Theorem. 16. The Eugenol's 3rd Zagreb polynomials is $x^2 + 8x + 3$.

Proof: Consider Eugenol's 3rd Zagreb polynomials $= ZG_3(G_3, x) = \sum_{uv \in E(G_1)} x^{|d_u - d_v|}$

$$= |n_{1,2}|x^{2-1} + |n_{1,3}|x^{3-1} + |n_{2,2}|x^{2-2} + |n_{2,3}|x^{3-2} + |n_{3,3}|x^{3-3}$$

$$= 2x + x^2 + 2x^0 + 6x + x^0 = x^2 + 8x + 3$$

$$\therefore ZG_3(G_3, x)(C_{10}H_{12}O_2) = x^2 + 8x + 3$$

Theorem. 17. The Eugenol's Forgotten Connectivity index is 132.

Proof: Consider Eugenol's forgotten Connectivity index $= \sum_{uv \in E(G_1)} [(d_u)^2 + (d_v)^2]$

$$= |n_{1,2}|[(1^2 + 2^2)] + |n_{1,3}|[(1^2 + 3^2)] + |n_{2,2}|[(2^2 + 2^2)] + |n_{2,3}|[(2^2 + 3^2)] + |n_{3,3}|[(3^2 + 3^2)]$$

$$= 2(5) + 10 + 2(8) + 6(13) + 18 = 10 + 10 + 16 + 78 + 18 = 132$$

$$\therefore F(C_{10}H_{12}O_2) = 132$$

Theorem. 18. The Eugenol's Forgotten polynomials is $x^{18} + x^{13} + x^{10} + 2x^8 + 2x^5$.

Proof: Consider Eugenol's forgotten polynomials $= \sum_{uv \in E(G_1)} x^{[(d_u)^2 + (d_v)^2]}$

$$= |n_{1,2}|x^{[1^2 + 2^2]} + |n_{1,3}|x^{[1^2 + 3^2]} + |n_{2,2}|x^{[2^2 + 2^2]} + |n_{2,3}|x^{[2^2 + 3^2]} + |n_{3,3}|x^{[3^2 + 3^2]}$$

$$= 2x^5 + x^{10} + 2x^8 + x^{13} + x^{18} = x^{18} + x^{13} + x^{10} + 2x^8 + 2x^5$$

$$\therefore F(C_{10}H_{12}O_2, x) = x^{18} + x^{13} + x^{10} + 2x^8 + 2x^5$$

Theorem. 19. The Eugenol's Symmetric division index is 26.3.

Proof: Consider Eugenol's Symmetric division index $= SDD(G_1)(C_{10}H_{12}O_2, x)$

$$SDD(G_1) = \sum_{uv \in E(G_1)} \left\{ \frac{\min(d_u, d_v)}{\max(d_u, d_v)} + \frac{\max(d_u, d_v)}{\min(d_u, d_v)} \right\}$$

$$= |n_{1,2}| \left\{ \frac{\min(1,2)}{\max(1,2)} + \frac{\max(1,2)}{\min(1,2)} \right\} + |n_{1,3}| \left\{ \frac{\min(1,3)}{\max(1,3)} + \frac{\max(1,3)}{\min(1,3)} \right\} + |n_{2,2}| \left\{ \frac{\min(2,2)}{\max(2,2)} + \frac{\max(2,2)}{\min(2,2)} \right\} + |n_{2,3}| \left\{ \frac{\min(2,3)}{\max(2,3)} + \frac{\max(2,3)}{\min(2,3)} \right\} + |n_{3,3}| \left\{ \frac{\min(3,3)}{\max(3,3)} + \frac{\max(3,3)}{\min(3,3)} \right\}$$

$$= 2 \left\{ \frac{1}{2} + 2 \right\} + \left\{ \frac{1}{3} + 3 \right\} + 2\{2\} + 6 \left\{ \frac{2}{3} + \frac{3}{2} \right\} + 2 = 2(2.5) + \frac{10}{3} + 4 + \frac{72}{6} + 2 = \frac{79}{6} = 26.3$$

$$\therefore SDD(G_1)(C_{10}H_{12}O_2, x) = 26.3$$

IV. CONCLUSION

We computed some Connectivity indices of Eugenol.

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