

Uncertain -Z Numbers based Face Sketch Recognition

Abdul Rahman¹, Sultan Alam² and Dr. Mohammadi Akheela Khanum³

^{1,2}School of Computing Science and Engineering, VIT Bhopal University, Bhopal, India

Email: abdulrahman@vitbhopal.ac.in, mohammadsultanalam@vitbhopal.ac.in

³School of Computer Science and Engg. & Information Science, Rajanukunte, Yelahanka, Bangalore India

Email: akheela.khanum@presidencyuniversity.in

Abstract— This research explores the fusion of fuzzy constraints and reliability to model human behaviors, referred to as Z-Numbers. However, Z-Numbers rely on specific information representation concepts, whereas it is widely acknowledged that interval concepts align better with our daily experiences. This motivation drives us to employ an enhanced version of Z-Numbers to address real-world challenges, such as the Sketching with Words (SWW) problem. SWW aims to automate the ranking of criminal face sketches in descending order of similarity, given that observer statements are fraught with perceptual uncertainties and fuzzy linguistic terms. Consequently, a dependable tool is imperative for practical problem-solving. The utilization of interval-valued complex fuzzy sets, centered on uncertain Z-Numbers, emerges as a promising approach to address such perception-based multicriteria group decision-making issues. In our proposed work, we incorporate uncertain Z-Numbers within the framework of interval-based fuzzy constraints and reliability. This approach aims to minimize the chances of criminals evading justice while safeguarding the rights of the innocent.

Index Terms— first term, fuzzy-geometry; Computing with word; Aggregation Operator; Z-Numbers

I. INTRODUCTION

In 2011, Zadeh [1] introduced the concept of Z-numbers as a powerful tool for representing uncertain information encountered in our daily lives. One practical application of Z-numbers is the sketching of a criminal's face based on eyewitness descriptions, which inherently involve uncertainty. While Z-numbers possess a natural structure, their practical applications have encountered certain limitations. To enhance the utility of Z-numbers in solving real-world problems, a reliable framework is essential. In pursuit of this goal, the seminal work of maximizing entropy was proposed [2] to construct an objective function for optimization. Additionally, various scholars have delved into related operations involving Z-numbers, including arithmetic operations, distance measures, similarity measures, and score functions. Aliev et al. [3], [4] defined operations for both discrete and continuous Z-numbers, while in author [5] introduced negation operations for discrete Z-numbers. Furthermore, Z-numbers have been integrated with other established theories, such as linguistic Z-numbers combined with the normal cloud model and the trapezium cloud model in [6] and [7], respectively. Bayesian networks have also been updated using Z-numbers [2]. Expanding on the advantages of Z-numbers in handling uncertainty, the VlseKriterijum-ska Optimizacija I Kompromisno Resenje (VIKOR) method was extended to Z-VIKOR in [8]. Moreover, Z-numbers were incorporated into the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) [9]. Furthermore, the concept of complex fuzzy sets (CFSs) was introduced by Ramot et al. [10], extending the notion.

of type-1 fuzzy sets to include a codomain within the complex plane. Despite this added dimension, the fundamental concept of fuzziness remains intact in CFSs [10, 11]. Alkouri and Salleh [12] introduced complex intuitionistic fuzzy sets (CIFSs) based on CFSs and IFSs. Additionally, in [13], Pythagorean and CFSs were combined to create complex Pythagorean fuzzy sets, and [14] introduced extended Z-numbers $ZE = (A, B), E$ with the inclusion of credibility factor E . The integration of Z-numbers with Atanassov's Intuitionistic Vague Imprecise Fuzzy Sets (AIVIFSs) was explored in [15]. In contrast, [16] introduced linguistic Z-numbers based on a polar coordinate system, while [17] proposed Z-Numbers Based on Complex Fuzzy Sets to address unmanned aerial vehicle purchasing problems. Furthermore, [18] presented a decision-making model based on Pythagorean fuzzy sets to solve Z-information problems.

The term "Sketching With Words" (SWW), coined by Beg et al. in [19], revolves around fuzzy geometric objects. SWW leverages concepts from f-geometry [20-21] and Computing With Words (CWW) [22]. In [23], SWW was applied to perception-based face recognition, aligning its computational framework with Z-numbers. Subsequently, in [24], SWW and Z-Numbers were integrated to create criminal face sketches, and [25] produced rankings of fuzzy similar faces using linguistic quantifiers such as "at least half," "as many as possible," and "most."

To enhance the ranking of fuzzy faces further, this paper employs the interval concept of fuzzy constraints and reliability on Z-numbers. This approach aims to reduce the chances of criminals evading justice and prevent the wrongful accusation of innocents. The remainder of this paper is organized as follows: Section 2 introduces the basic concepts of f-geometry, Z-numbers, SWW, and Complex Fuzzy Sets (CFS). In Section 3, we explain Enhanced Postulate Improved Z-numbers (IVZ) and Interval-valued Complex Fuzzy Sets (IVZCFS). Section 4 details the algorithm, while Section 5 discusses applications. Section 6 presents experimental work and results, and the final section includes conclusions and future directions.

II. PRELIMINARIES

Zadeh's concept of 'f-geometry' [1] represents an endeavor to formalize the cognitive capacity of the human brain for measuring and manipulating information in the context of machine-based operations. F-geometry is an exemplar of "Flu," which stands for Fuzzy Logic Uncertainty. Flu hinges on the foundation of fuzzy validity (f-validity) reasoning, a tool that emulates the remarkable human ability to perform a diverse array of physical and mental tasks without relying on precise measurements or extensive computations. Within f-geometry, various types of f-objects, such as fuzzy lines, fuzzy circles, fuzzy semicircles, and fuzzy triangles, are employed to represent their crisp geometry counterparts, including lines, circles, semicircles, triangles, rhombuses, squares, and rectangles, as depicted in Fig.3 for reference. F-geometry serves as a counterpart to Euclidean geometry within the realm of crisp (non-fuzzy) theory. In f-geometry, shapes are sketched freehand, devoid of conventional geometric instruments. The distinctive facial features of foreheads, eyes, noses, lips, and chins are conceptualized as fuzzy semicircles (f-semicircles), fuzzy circles (f-circles), fuzzy triangles (f-triangles), fuzzy circles (f-circles), and fuzzy semicircles (f-semicircles), respectively. These representations are visually illustrated in Figures 1 and 2.



Fig. 1 Different types of nose and lips /chins as f-triangle and f-semicircle respectively



Fig 2. Different types of forehead and eyes as f-semicircle and f-circle respectively.



Fig. 3 Different types of f-objects like fuzzy line, fuzzy circle, fuzzy semicircle, and fuzzy triangle



Fig 4. Outlines of a face is made of different f-geometric objects



Fig. 5 Membership function of A and the underlying possibility distribution of B

A. Sketching with Words and Z- Number

The concept of "Sketching with Words" (SWW), often abbreviated as 'SWW,' draws its inspiration from both Computing with Words (CWW) and f-geometry. While traditional computing predominantly involves the manipulation of numbers and symbols, CWW introduces a methodology wherein the fundamental units of computation are words and propositions extracted from natural language. For instance, terms like "small eye," "long nose," or "not fairly long chin" [25] become integral elements of the computational process.

In SWW, these linguistic entities, referred to as 'f-objects,' play a pivotal role in constructing the facial sketches of criminals. Furthermore, specific facial features, including foreheads, eyes, noses, lips, and chins, are portrayed as f-semicircles, f-circles, f-triangles, f-circles, and f-semicircles, respectively. Fig. 4. Outlines of a face is made of different f-geometric objects. and symbols, language.

A "Z-number" is essentially a fusion of fuzzy numbers, denoted as (A, B) . In the context of this study, the variables 'A' and 'B' are replaced with appropriate mathematical functions. A "Z-valuation" is essentially a measurement or assignment statement in the form of "X is (A, B)." Here, 'A' represents the possibility distribution of X, while 'B' is an interpreted probability value associated with a fuzzy event. The first component, 'A,' acts as a fuzzy restriction denoted as $R(X)$, defining the range of values that X can assume, expressed as "X is A," where 'A' represents a fuzzy set. When X represents a random variable, the probability distribution of X serves as a "probabilistic restriction" on X [1]. Please see Fig. 5.

Perception-based natural language statements can be expressed as "Z-valuations," which are ordered triples (X, A, B) and are treated as assignment statements, such as "X is (A, B)." Simple examples of Z-valuations include statements like "he has small eyes, approximately 2.8 cm in size, very likely," "forehead of the criminal, small, uncertain," or "his nose, small, highly probable."

The parameters 'A' and 'B' within Z-numbers represent the fuzzy constraint and reliability, respectively. These two components are linked through an underlying possibility distribution (UPD). Determining the UPD is typically a challenging task. In previous work [1], compatibility and consistency were considered to calculate the UPD. However, this calculation process involves solving a nonlinear programming problem, which can lead to complex computations, as illustrated in Figure 1. In this context, μ_A (representing possibilistic constraints) serves as the possibility distribution of X constrained by a fuzzy relation, while p_X represents the underlying (hidden) probability density of X.

To enhance the practical utility of Z-numbers, it is essential to develop a dependable approach. In our proposed work, we advance Z-numbers to incorporate uncertain linguistic Z-numbers. The transformation method between uncertain linguistic Z-numbers and interval-valued complex fuzzy sets (IVCFs) is detailed in [27].

B. Complex Fuzzy Set (CFS)

CFS is denoted as S , defined over the domain U , can be represented as $S = \{(x, \mu_S(x)) | x \in U\}$, where the membership function $\mu_S(x)$ takes the form of complex-valued numbers $\mu_x = r_s(x) \cdot e^{j\omega_s(x)}$ [14]. Here, the values of $\mu_S(x)$ reside in the range $[0, 1]$, and the phase angle $\phi_S(x)$ belongs to the interval $[0, 2\pi]$.

III. ENHANCED POSTULATE

A. Improved Z-numbers (IVZ)

Considering that an interval concept is more appropriate than a specific concept to characterize the fuzzy constraint and reliability, Peng et al. [17]. Z-numbers were combined with AIVIFSs to transform it into 'uncertain Z-numbers'.

IVZ = ((very high, [0.5 0.7], (sure, [0.7, 0.9])). Expression can be represented by pair of bounded interval IVZ=([S_{a-}, S_{a+}], [H_{b-}, H_{b+}]), where S_{a-} and S_{a+} represent the upper and lower bounds of linguistic constraint, respectively. H_{b-} and H_{b+} indicate the upper and lower bounds of reliability, respectively.

B. Inter valued Complex Fuzzy Sets (IVCFSs)

The aim of IVZ is to better represent the fuzzy constraint and reliability. IVCFSs can achieve the aim [20], the desired expression is {anticipated budget deficit, between 1.8 and 2.2 million dollars, likely, very likely}. IVCFSs can be characterized by J_s as $J_s = [r^-_s(x), r^+_s(x)] * e^{j[\omega^-_s(x), \omega^+_s(x)]}$ $r^-_s(x)$ and $r^+_s(x)$ represent the upper and lower bounds of amplitude term, respectively. $\omega^-_s(x)$ and $\omega^+_s(x)$ indicate the upper and lower bounds of phase term, respectively.

IV. ALGORITHM

An MCGDM method for Z-information is presented based on the proposed operations. Algorithm Process MCGDM has always been closely related to daily life.

Step 1: Conversion of Evaluation Information into Corresponding IVCFS

Like Z-numbers, uncertain Z-numbers exhibit ever-changing distributions for their two components because X is an uncertain variable, not a random one. When natural language is employed to describe these two components of Z-numbers, the implications of A and B become embedded in the corresponding function and distribution, as conveyed through linguistic information [1]. Thus, when linguistic variables in uncertain Z-numbers are transformed into Interval-Valued Complex Fuzzy Sets (IVCFSs), the inherent fuzziness of [S_{a-}, S_{a+}] and the randomness of [H_{b-}, H_{b+}] can be embodied using $r^-_s(x)$ and $r^+_s(x)$ and $\omega^-_s(x)$ and $\omega^+_s(x)$.

Definition 1: Given two Linguistic Term Set S and H,

where $S = \{S_0, S_1, \dots, S_E | E \in N\}$ and $H = \{H_0, H_1, \dots, H_F | F \in N\}$,

for $([S_{a-}, S_{a+}], [H_{b-}, H_{b+}])$, and $J_s = [r^-_s(x), r^+_s(x)] * e^{j[\omega^-_s(x), \omega^+_s(x)]}$ (1)

Employ Definition 1 to convert the evaluation information into the corresponding IVCFS using $r^-_s(x)$, $r^+_s(x) \in [0, 1]$ and $\omega^-_s(x)$, $\omega^+_s(x) \in [0, 2\pi]$

Step 2: Calculation of the Criteria Weight Vector In general, methods for calculating weights can be categorized into two aspects: subjective and objective. Subjective methods often rely on expert judgments, but their objectivity is questionable. Therefore, many scholars prefer employing objective methods. Among these, the entropy weight method (EWM) is widely accepted due to its high reliability and convenience [23][24].

a. Calculate Generalized Entropy

Entropy, a fundamental concept in thermodynamics and information theory, is typically used to quantify the disorder in a system. Numerous entropy calculation methods have been proposed for classical fuzzy sets [19]. However, to our knowledge, there is currently no approach for calculating the entropy of Z-numbers from the perspective of Complex Fuzzy Sets (CFSs). The generalized entropy, denoted as $E_g(J)$, is defined by equation (2):

b. Calculate the Power-Weighted Average of Interval-Valued Complex Fuzzy Sets

Weight Calculation: For the aggregation operator presented in equation 2,

$$E_g(J) = 1 - \left[\frac{1}{2} (|r^-_s(x) + r^+_s(x) - 1|^p + \frac{1}{2\pi} |\omega^-_s(x) + \omega^+_s(x) - 2\pi|^p) \right]^{1/p} \quad (2)$$

Definition 2: Given $J_s = [r^-_s(x), r^+_s(x)] * e^{j[\omega^-_s(x), \omega^+_s(x)]}$ $\{i = 1, 2, \dots, n\}$, the weight vector $\{w_1, w_2, \dots, w_n\}$ is derived as

$$w_i = \frac{1 - E_g(J_i)}{\sum_{k=1}^n [1 - E_g(J_k)]} \quad (3)$$

Step 3 Interval-Valued Complex Fuzzy Sets Power Weighted Average (IVCFSsPWA)

We introduce the concept of Interval-Valued Complex Fuzzy Sets Power Weighted Average (IVCFSsPWA), which plays a crucial role in the information fusion process. Let J_s represent a set of Interval-Valued Complex Fuzzy Sets defined as

$$J_s = [r^-_i(x), r^+_i(x)] * e^{j[\omega^-_i(x), \omega^+_i(x)]} \quad \{i=1, 2, 3, \dots, n\}.$$

The integrated J, denoted as J, is computed using the following equation (4):

$$J = IVCFSSPWA(J_1, J_2, \dots, J_n) = J_1 \oplus J_2 \oplus \dots \oplus J_n \quad (4)$$

The Integration Operation, as defined by equation (5), allows us to combine these sets effectively:

$$J_1 \oplus J_2 = [1 - \prod_{i=1}^n (1 - r_i^-)^{w_i}, 1 - \prod_{i=1}^n (1 - r_i^+)^{w_i}] \cdot e^{j[2\pi(1 - \prod_{i=1}^n (1 - (\omega_i^-/2\pi))^{\omega_i}), 2\pi(1 - \prod_{i=1}^n (1 - (\omega_i^+/2\pi))^{\omega_i})]} \quad (5)$$

Step 4 & 5: Distance Measurement and Ranking of Alternatives

To assess the differences and similarities between J_1 and J_2 , we use equations (6) and (7) for distance and similarity measurement, respectively:

$$D(J_1, J_2) = \frac{1}{4} (|r_1^-(x) - r_1^+(x)| + |r_2^-(x) - r_2^+(x)| + |\frac{\omega_1^-(x) - \omega_1^+(x)}{2\pi}| + |\frac{\omega_2^-(x) - \omega_2^+(x)}{2\pi}|) \quad (6)$$

$$S(J_1, J_2) = 1 - \frac{1}{\sqrt{2}} \left(\left(\frac{r_1^-(x) + r_1^+(x)}{2} - \frac{r_2^-(x) + r_2^+(x)}{2} \right)^2 + \left(\frac{\omega_1^-(x) + \omega_1^+(x)}{4\pi} - \frac{\omega_2^-(x) + \omega_2^+(x)}{2\pi} \right)^2 \right)^{1/2} \quad (7)$$

V. APPLICATION

The application of Z-numbers has already been demonstrated in [27] to generate criminal face sketches based on eyewitness perception. Z-numbers are particularly useful in scenarios where onlookers are bound by specific fuzzy constraints within uncertain environments. Since facial description often involves Multicriteria Group Decision Making (MCGDM), an interval-based approach is adopted to address uncertainty and facilitate more accurate representation of fuzzy constraints. Uncertain Z-numbers (IVZ) extend support for Interval Valued Concepts, which, in turn, align linguistic descriptions more closely with everyday expressions[28] . Consequently, the utilization of IVZ encourages the generation of a list of face sketches, ranked in decreasing order of similarity, which could prove invaluable in apprehending suspects. In this context, IVZ captures both fuzziness and uncertainty through [Sa] and [Hb], respectively. The sets S and H are defined as follows: S = {s0 = extremely small, s1 = very small, s2 = small, s3 = slightly small, s4 = slightly large, s5 = large, s6 = very large, s7 = extremely large}, H = {h0 = extremely uncertain, h1 = very uncertain, h2 = uncertain, h3 = slightly uncertain, h4 = slightly certain, h5 = certain, h6 = very certain, h7 = extremely certain}.

VI. EXPERIMENTS AND RESULT

Following the computation of the criteria weight vector for eyewitnesses e1, e2, and e3, the estimated collective evaluation information using the aggregation operator is presented in Tables 5, 6, and 7. Furthermore, Tables 8(a) and 8(b) depict the calculated distances between alternatives and ideal solutions, namely D_p and D_n . In the experimental phase, the input consists of a 'query statement' submitted by the onlooker(s), which serves as a description of facial image1. The relevance of the input image to other stored images is estimated using a relevance matrix. The ranking of different faces is depicted in Figure 6, presented in ascending order of f-similarity, wherein a lower ranking number signifies a higher level of relevance against the input decreasing order.

TABLE I. INPUTS ARE IN LINGUISTIC VARIABLE

		c1(forehead)	c2(nose)	c3(eyes)	c4(lips and chin)
eyewitness e1	f1	{ slightly small, slightly large, very uncertain, certain }	{ very small, very large, slightly certain, certain }	{ small, slightly small, uncertain , slightly uncertain }	{ slightly large, large, certain, extremely certain }
	f2	{ very small, small, extremely uncertain, slightly certain }	{ large, extremely large slightly certain, very certain }	{ small, slightly small, uncertain, certain }	{ slightly small, large, slightly certain, extremely certain }
	f3	{ small, slightly small, very uncertain, uncertain }	{ large,, very large, very uncertain, certain }	{ small, slightly small, slightly uncertain , slightly certain }	{ slightly large, large , very uncertain, uncertain }
	f4	{ slightly small, large, slightly uncertain, certain }	{ very small, large, very3uncertain,, uncertain }	{ small, slightly small, uncertain , slightly uncertain }	{ slightly small, , slightly large slightly uncertain, slightly certain }
eye	f1	{ slightly small, slightly large, extremely uncertain, very certain }	{ small, slightly small, slightly certain, extremely certain }	{ small, slightly small, uncertain slightly uncertain }	{ large, very large, very uncertain, uncertain }

eyewitness e1	f2	{ very small, small, uncertain, slightly uncertain }	{ slightly large, , extremely large, uncertain, slightly certain }	{ very small, slightly large, uncertain slightly uncertain }	{ small, , slightly large, slightly uncertain, certain }
	f3	{ very small, extremely large, very uncertain, slightly uncertain }	{ slightly large, , extremely large, slightly uncertain, slightly certain }	{ very small, small, uncertain slightly uncertain }	{ small, slightly small, slightly uncertain, certain }
	f4	{ very small, small, slightly certain, extremely certain }	{ small, very large, very certain, extremely certain }	{ very small, slightly small, uncertain slightly uncertain }	{ small, slightly small, uncertain, certain }
	f1	{ slightly small, slightly large, very uncertain, certain }	{ slightly-large, large, slightly uncertain, certain }	{ very small, slightly large, slightly certain, extremely certain }	{ slightly-large, extremely large, slightly certain, certain }
	f2	{ slightly small, small, slightly certain, extremely certain }	{ small, slightly-large, , slightly uncertain, slightly certain }	{ small, extremely large, very uncertain , slightly uncertain }	{ small, slightly large, very uncertain, certain }
	f3	{ small, slightly large, slightly uncertain, extremely certain }	{ very small, small, slightly uncertain, certain }	{ slightly small, extremely large, slightly certain, uncertain }	{ small, extremely large, very certain, extremely certain }
	f4	{ small, slightly small, slightly uncertain, certain }	{ large,, very large, uncertain, slightly uncertain }	{ very small, slightly large, uncertain slightly uncertain }	{ slightly- small, slightly large, uncertain, certain }

TABLE II INPUTS ARE IN IVCFCFS FORM FOR EYEWITNESS E1

	c1	c2	c3	c4
f ₁	(0.43,0.57,0.90,4.49)	(0.14,0.86,3.59,4.49)	(0.29,0.43,1.79,2.69)	(1.79,2.24,4.49,6.28)
f ₂	(0.14,0.29,1.79,2.69)	(0.71,1,3.59,5.38)	(0.29,0.57,1.79,4.49)	(0.43,0.71,3.59,6.28)
f ₃	(0.29,0.43,0.90,1.79)	(0.71,0.86,0.90,4.49)	(0.71,1,2.69,3.59)	(0.57,0.71,0.90,1.79)
f ₄	(0.43,0.57,2.69,4.49)	(0.14,0.71,0.90,1.79)	(0.86,1,1.79,2.69)	(0.43,0.57,2.69,3.59)

TABLE III. INPUTS ARE IN IVCFCFS FORM EYEWITNESS E2

	c1	c2	c3	c4
f1	(0.28,1,2.690,6.28)	(0.42,0.57,0.89,4.48)	(0.14,0.85,1.79,3.49)	(1.39,0.854,3.58,3.58)
f2	(0.14,0.28,1.79,3.58)	(0.14,0.28,1.79,2.69)	(0.710,1,1.79,2.69)	(0.73,1,1.79, 2.62)
f3	(0.14,1,0.89,3.58)	(0.28, 0.42, 0.89, 1.79)	(0.714,0.85,2.69,1.79)	(0.71,0.89,0.90,1.79)
f4	(0.14,0.28, 3.58, 6.28)	(0.42,0.57,2.69,4.49)	(0.14,0.71,1.79,4.49)	(0.14,0.71,2.69,4.48)

TABLE IV INPUTS ARE IN IVCFCFS FORM EYEWITNESS E3

	c1	c2	c3	c4
f1	(0.43,0.57,0.90,4.49)	(0.14,0.86,3.59,4.49)	(0.29,0.43,1.79,2.69)	(1.79,2.24,4.49,6.28)
f2	(0.14,0.29,1.79,2.69)	(0.71,1,3.59,5.38)	(0.29,0.57,1.79,4.49)	(0.43,0.71,3.59,6.28)
f3	(0.29,0.43,0.90,1.79)	(0.71,0.86,0.90,4.49)	(0.71,1,2.69,3.59)	(0.57,0.71,0.90,1.79)
f4	(0.43,0.57,2.69,4.49)	(0.14,0.71,0.90,1.79)	(0.86,1,1.79,2.69)	(0.43,0.57,2.69,3.59)

TABLE V IVCFCFS FOR E₁

Face	IVCFCFS
f1	(0.49,1,3.16,6.28)
f2	(0.32,1,2.08,3.45)
f3	(0.27,1,1.64,6.28)
f4	(0.208,0.64,4.50,6.28)

TABLE VI IVCFCS FOR E2

Face	IVCFCS
f1	(0.43,0.69,3.65,6.28)
f2	(0.49,1,3.17,6.28)
f3	(0.55,1,1.24,2.71)
f4	(0.58,1,1.50,2.60)

TABLE VII IVCFCS FOR E3

Face	IVCFCS		c1	c2	c3	c4
f1	(0.40,1,2.77,6.28)	e1	0.32	0.24	0.45	0.43
f2	(0.22,1,2.57,6.28)	e2	0.27	0.44	0.36	0.35
f3	(0.29,1,4.3,6.28)	e3	0.28	0.34	0.25	0.47
f4	(0.48,0.68,12.06,3.90)					

TABLE VIII (A) & (B) DISTANCE DN DP

	c1	c2	c3	c4
e1	0.38	0.49	0.73	0.73
e2	0.52	0.61	0.50	0.28
e3	0.48	0.45	0.40	0.54



Fig 6. Ranking of facial images in non-decreasing order of Similarity

VII. CONCLUSION AND FUTURE DIRECTIONS

The integration of Complex Fuzzy Sets has significantly enhanced the performance of Z-Numbers, a relatively new concept that is still in its early stages of development. Complex Fuzzy Sets have demonstrated their versatility, particularly in the realm of Multi-Criteria Decision Making. Moreover, their applicability extends beyond the mere ranking of criminal face sketches; they offer a robust platform for interpreting and expressing daily human behaviors and habits.

REFERENCES

- [1] Zadeh L.A, A note on Z-numbers . J Inf Sci 181(14):2923–2932(2011)
- [2] Zadeh L.A Fuzzy sets and information granularity. In: Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems. pp 433–448. https://doi.org/10.1142/9789814261302_0022(1996).
- [3] Zadeh L.A ,From fuzzy logic to extended fuzzy logic—the concept of f-validity and the impossibility principle, plenary session, FUZZ-IEEE 2007; Imperial College, London, UK (2007)
- [4] Zadeh L.A ,From computing with numbers to computing with words—from manipulation of measurements to manipulation of perceptions. IEEE Trans Circuits Syst I 45:105–119 (1999)
- [5] Qianlei Jia , Jiayue Hu , and Enrique Herrera-Viedma, “A Novel Solution for Z-numbers Based on Complex Fuzzy Sets and Its Application in Decision-Making System,” IEEE Transactions On Fuzzy Systems, Vol. 30, No. 10, October 2022, pp. 4102–4114, 2022.
- [6] T. Verma and A. Kumar, “Ambika methods for solving matrix games with Atanassov’s intuitionistic fuzzy payoffs,” IEEE Trans. Fuzzy Syst., vol. 26, no. 1, pp. 270–283, Feb. 2018.
- [7] Rahman A, Beg MMS (2013) Estimation of f-validity of geometrical objects with OWA operator weights. In: Fuzz-2013 IEEE international conference on fuzzy systems on July 7–10
- [8] Rahman A, Beg MMS (2014) Towards aggregation in fuzzy objects. In: Proceeding of 5th international conference on computer and communication technology ICCCT-2014, September 26–28 Allahabad India, pp 373–380
- [9] Imran BM, Beg MMS , Elements of sketching with words (an extended paper). Int J Granul Comput Rough Sets Intell Syst (Inderscience Publishers, (UK). ISBN:978-1-4244-7964-1. pp 241–246. <https://doi.org/10.1109/GrC.2010.47> (2018)
- [10] Imran BM, Beg MMS, Fuzzy identification of geometric shapes. In: International conference on computer science and information technology 2012, Bangalore (2012)
- [11] Imran BM, Beg MMS , Towards computational forensics with f-geometry. In: World conference on soft computing, 2011; San Francisco State University, California, USA(2011)
- [12] Imran BM, Beg MMS (2018) Fuzzy towards perception based image retrieval. In: International conference on computer science and information technology, Bangalore

- [13] Yang X, He Y, Dong L, Zhao H Particle swarm optimization for determining fuzzy measures from data. *Inf Sci* 181(9):4230–4252 (2011)
- [14] D. Ramot, R. Milo, M. Friedman, and A. Kandel, “Complex fuzzy sets,” *IEEE Trans. Fuzzy Syst.*, vol. 10, no. 2, pp. 171–186, Apr. 2002.(2002)
- [15] Rahman A, Beg MMS , Estimation of f-validity of geometrical objects with OWA operator weights, an extended paper. In: Special issue of BIJIT on fuzzy logic of BVICAM’s International Journal of Information Technology, (BIJIT) ISSN 345 0973–5658, vol 6. pp 663–665(2014)
- [16] Rahman A, Beg MMS (2013) Investigation of OWA operator weights for estimating f-validity of geometrical shapes. In: Third annual world conference on soft computing, (WCSC-2013).
- [17] W. Jiang, Y. Cao, and X. Deng, “A novel Z-network model based on Bayesian network and Z-number,” *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 8, pp. 1585–1599, Aug. 2020.
- [18] K . W. Shen and J. Q. Wang, “Z-VIKOR method based on a new comprehensive weighted distance measure of Z-number and its application,” *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3232–3245, Dec. 2018
- [19] F . Sabahi, “FN-TOPSOS: Fuzzy networks for ranking traded equities,” *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 2, pp. 315–332, Apr. 2017.
- [20] H. G. Peng, H. Y. Zhang, J. Q Wang, and L. Li, “An uncertain Z-number multicriteria group decision-making method with cloud models,” *Inf. Sci.*, vol. 501, pp. 136–154, 2019.
- [21] A. S. Alkouri and A. R. Salleh, “Complex intuitionistic fuzzy sets,” in *Proc. Int. Conf. Fundam. Appl. Sci.*, 2012.
- [22] S. Dick and O. Yazdanbakhsh, “A systematic review of complex fuzzy sets and logic,” *Fuzzy Sets Syst.*, vol. 338, pp. 1–22, 2017.
- [23] Q. Jia, J. Hu, Q. He, W. Zhang, and E. Safwat, “A multicriteria group decision-making method based on AIVIFSs, Z-numbers, and trapezium clouds,” *Inf. Sci.*, vol. 566, pp. 38–56, 2021.
- [24] Abdul Rahman, Tanvir Ahmad, M.M.S. Beg. “Ranking of Fuzzy Similar Faces By Using Ordered Weighted Aggregation Operator and Relevance Matrix” , published in International Conference on Soft Computing and Software Engineering (SCSE 2015) 6-7 March, 2015, UC Berkley, USA.Elsevier,2015.
- [25] Abdul Rahman, M.M.S. Beg, “Face Sketch Recognition- An Application of Z-Numbers “, published in Springer International Journal of Information Technology, (BIJIT) ISSN 0973-5658
- [26] Abdul Rahman, M.M.S. Beg, “Face Sketch Recognition Using Sketching With Words”, Published in Springer International Journal of Machine Learning and Cybernetics Int. J. Mach. Learn. & Cyber. DOI 10.1007/s13042-014-0256-y Springer 2014 ISSN 1868-8071.
- [27] J. Ye, “Multicriteria fuzzy decision-making method using entropy weights based correlation coefficients of interval-valued intuitionistic fuzzy sets,” *Appl. Math. Model.*, vol. 34, no. 12, pp. 3864–3870, 2010
- [28] J. Ye, “Multicriteria fuzzy decision-making method using entropy weights based correlation coefficients of interval-valued intuitionistic fuzzy sets,” *Appl. Math. Model.*, vol. 34, no. 12, pp. 3864–3870, 2010