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A Note on Minimum Degree Index for Graphs

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***Abstract*— In research work, introduces the notion of the new topological index called Minimum Degree Index (MDI), for a graph G and obtain MDI for few graphs. Also, we obtain MDI for few chemical structures.**

I. INTRODUCTION

In our paper, the graphs we refer are always simple, finite, undirected and connected. We symbolize the vertex set as V(G)and the edge set as E(G). Let d_G (v_i) denote the degree of vertex vi. For notations and standard terminologies, we follow the textbook from [9]. The graph energy and topological index of a graph are very attractive due to their importance in mathematical chemistry. Topological indices play a significant role in the research of drug development. A huge variety of definitions in the field of topological indices and technology in drug discovery are introduced. There is many topological indices, based on distance and degree of vertex for graphs. It is a numerical value obtained from the structure of graph. Several graph indices have been defined by using vertex degree concept and applied for the study of theoretical chemistry [18]. In [10]-[17] many graph indices and their applications in various disciplines are studied.
Motivated by the above works, we introduce the new topological indices called Minimum Degree Index (MDI). In this paper we compute Minimum Degree Index for a few special graphs. In section 2 we give necessary fundamental definitions, theorems and results we use in the construction of paper. In section 3 we compute MDI for a few graphs. In section 4 we compute Minimum Degree Index for few chemical structures.

II. PRELIMINARIES

As per the requirement of this paper we consider the following definitions and theorems from the papers of V. R. Kulli [22] [19].
Definition 2.1. For a graph G, the elliptic delta index is given as follows,

$$E\Delta(G) = \sum_{u,v\in E(G)} (\Delta_u + \Delta_v)\sqrt{\Delta_u^2 + \Delta_v^2}.$$

Definition 2.2. For a graph G, the modified elliptic delta index is given as follows

$$^{m}E\Delta(G) = \sum_{u,v\in E(G)} \frac{1}{(\Delta_u + \Delta_v)\sqrt{\Delta_u^2 + \Delta_v^2}}$$

From [22] we have the following results.
Theorem 2.3. *For a graph G of silicate networks SL_n. We have,*

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$$E\Delta(G) = (90\sqrt{17} + 576\sqrt{2})n^2 + (30\sqrt{17} - 372\sqrt{2})n.$$

Theorem 2.4. For a graph G of silicate networks SL_n. We have

$$^m E\Delta(G) = \frac{6n}{2\sqrt{2}} + \frac{18n^2 + 6n}{5\sqrt{17}} + \frac{18n^2 - 12n}{32\sqrt{2}}$$

Theorem 2.5. For a graph G of honeycomb networks HC_n, we have,

$$E\Delta(G) = 72\sqrt{2}n^2 + (36\sqrt{5} - 120\sqrt{2})n + 60\sqrt{2} - 36\sqrt{5}.$$

Theorem 2.6. For a graph G of honeycomb networks HC_n. We have,

$$^m E\Delta(G) = \frac{6n}{2\sqrt{2}} + \frac{12n - 12}{3\sqrt{5}} + \frac{9n^2 - 15n + 6}{8\sqrt{2}}$$

Theorem 2.7. For a graph G of Oxide networks OX_n, we have,

$$E\Delta(G) = 162\sqrt{2}n^2 + (24\sqrt{10} - 54\sqrt{2})n.$$

Theorem 2.8. For a graph G of Oxide networks $OX_{n,}$ we have,

$$^m E\Delta(G) = \frac{6n}{4\sqrt{10}} + \frac{9n^2 - 3n}{18\sqrt{2}}.$$

Theorem 2.9. For a graph G of hexagonal networks HX_n. We have,

$$E\Delta(G) = 288\sqrt{2}n^2 + (144\sqrt{5} - 1008\sqrt{2})n + 30\sqrt{17} - 252\sqrt{5} + 816\sqrt{2}.$$

Theorem 2.10. For a graph G of hexagonal networks HX_n. We have,

$$^m E\Delta(G) = \frac{12}{3\sqrt{5}} + \frac{6}{5\sqrt{17}} + \frac{6n - 18}{8\sqrt{2}} + \frac{12n - 24}{12\sqrt{5}} + \frac{9n^2 - 33n + 30}{32\sqrt{2}}$$

III. Computing of Minimum Degree Index for few graphs

Let us consider G be a finite, simple graph having $\{v_{1,}v_{2,}v_{2,} \dots \dots . v_{n,}\}$ as vertex set and d(u_i) and d(v_i) denote the degree of u_i and v_i correspondingly for, i =1, 2, 3,...,n. Then, Minimum Degree Index (MDI) is given by.

$$\text{MDI(G)} = n^2\left(\sum_{u,v\in E} \min[d(u_i), d(v_i)]\right) \qquad (1)$$

We have the following computations, Using the definition of Minimum Degree Index(MDI)

- For Path P_n of n vertices,

$$\text{MDI}(P_n) = 2n^3 - 4n^2$$

- For Cycle Cn of n vertices,

$$\text{MDI}(C_n) = 2|E| = 2n3$$

- For complete graph K_n of n vertices,

$$\text{MDI}(K_n) = n^3 \frac{(n-1)2}{2}$$

- For fan graph F_n of n vertices,

$$\text{MDI}(F_n) = 6n^3 - 7n^2$$

- For Star graph S_n of n vertices,

$$\text{MDI}(S_n) = n^4 - 2n^3 + n^2$$

- For Double Star graph $S_n \times S_n$ of n vertices,

$$\text{MDI}(S_n \times S_n) = n^3 + 2n^2$$

- For graph Petersen of n vertices,

$$\text{MDI}(P(n,k)) = 30n2$$

IV. SOME RESULTS INVOLVING CHEMICAL STRUCTURE OF CERTAIN NETWORKS

A. Minimum Degree Index for Silicate Network

Silicates network is a structure of ring formed by the connected structures of tetrahedrons. It is the most complicated class of mineral. It is the most largest mineral found in the earth crust and the most interesting mineral found so far. It is chemically represented as SiO_4. In a two dimensional plane, a silicate sheet is formed by connecting tetrahedrons to form a ring, linked by shared oxygen nodes with other rings which produces a sheet-like structure.

The count of silicon nodes equals the product of count of nodes of HC_n, and hence count of edges of SL_n is $6 \times 6n^2 = 36n^2$

we have,

$$E_{3,3} = \{u_i v_i \epsilon E(G) \,|d_G(u_i) = d_G(v_i) = 3\}, |E_{3,3}| = 6n.$$

$$E_{3,6} = \{u_i v_i \epsilon E(G) \,|d_G(u_i) = 3, d_G(v_i) = 6\}, |E_{3,6}| = 18n^2 + 6n.$$

$$E_{6,6} = \{u_i v_i \epsilon E(G) \,|d_G(u_i) = d_G(v_i) = 6\}, |E_{6,6}| = 18n^2 - 12n.$$

Theorem 4.1. MDI for silicate network SL_n. Then,

$$\text{MDI}(SL_n) = 162n^4 - 36n^3$$

Proof. By definition we have,

$$\text{MDI}(SL_n) = n^2\left(\textstyle\sum_{u,v\in E} \min[d(u_i), d(v_i)]\right) \qquad (2)$$

$=162n^4 - 36n^3$

Also we have the following results for SL_n

(i) $\text{MDI}(G) > E\Delta(G)$

i.e., $162n^4 - 36n^3 > (90\sqrt{17} + 576\sqrt{2})n^2 + (30\sqrt{17} - 372\sqrt{2})n.$

(ii) $\text{MDI}(G) >^m E\Delta(G)$

i.e., $162n^4 - 36n^3 > \frac{6n}{2\sqrt{2}} + \frac{18n^2+6n}{5\sqrt{17}} + \frac{18n^2-12n}{32\sqrt{2}}.$

B. Minimum Degree Index for Honeycomb Network

In graph theory, a honeycomb network is a regular network with a hexagonal topology that can be constructed in any dimension. Honeycomb networks are often used to study complex systems and their properties. Honeycomb networks are used in various of applications such as, Physics, Computer science, Materials science, and Communication network design. It is represented as HC_n.

In [22] obtained that the graph G has $6n^2$ vertices and $9n^2 - 3n$ edges. In HC_n there are three types of edges as follows:

$$E_{2,2} = \{u_i v_i \epsilon E(G) \,|d_G(u_i) = d_G(v_i) = 2\}, |E_{3,3}| = 6.$$

$$E_{2,3} = \{u_i v_i \epsilon E(G) \,|d_G(u_i) = 2, d_G(v_i) = 3\}, |E_{3,6}| = 12n - 12.$$

$$E_{6,6} = \{u_i v_i \epsilon E(G) \,|d_G(u_i) = d_G(v_i) = 6\}, |E_{6,6}| = 9n^2 - 15n + 6.$$

Theorem 4.2 MDI for honeycomb network HC_n, is given by,

$$\text{MDI}(G) = 27n^4 - 21n^3 + 6n^2.$$

Proof. By definition we have,

$$\text{MDI}(HC_n) = \text{n}^2\left(\sum_{u,v\in E} min[d(u),d(v)]\right) \quad (3)$$

$$= 27\text{n}^4 - 21\text{n}^3 + 6\text{n}^2.$$

Also we have the following results for HC_n

(i) MDI(G) > Eδ(G)

i.e., $27n^4 - 21n^3 + 6n^2 > 72\sqrt{2}n^2 + (36\sqrt{5} - 120\sqrt{2})n + 60\sqrt{2} - 36\sqrt{5}$.

(ii) MDI(HC_n) > ^{m}Eδ(HC_n)

i.e., $27n^4 - 21n^3 + 6n^2 > \frac{6}{2\sqrt{2}} + \frac{12n-12}{3\sqrt{5}} + \frac{9n^2-15n+6}{8\sqrt{2}}$.

C. Minimum Degree Index for Oxide Network

An oxide network is a network that is created by removing all the silicon nodes from a silicate network. An n-dimensional oxide network is represented by OX_n. Oxide networks are important for studying the metric and structural properties of silicate networks. The diameter of both SL_n and OX_n is the same.
In an oxide network OX_n there exists 9n2 + 3n vertices and 18n2 edges. Based on degrees and end vertices each edge as follows,

$$E_{2,4} = \{u_iv_i\epsilon E(G)\ |d_G(u_i) = 2, d_G(v_i) = 4\}, |E_{3,3}| = 12n.$$

$$E_{4,4} = \{u_iv_i\epsilon E(G)\ |d_G(u_i) = d_G(v_i) = 4\}, |E_{3,6}| = 18n^2 - 12n.$$

Theorem 4.3. MDI for Oxide Network OX_n. , is given by,,,

$$\text{MDI}(OX_n) = 72\text{n}^4 - 24\text{n}^3.$$

Proof. By definition we have,

$$\text{MDI}(OX_n) = \text{n}^2\left(\sum_{u,v\in E} min[d(u),d(v)]\right) \quad (4)$$

$$= 72\text{n}^4 - 24\text{n}^3.$$

Also we have the following results for OX_n

(i) MDI(OX_n) > EΔ(OX_n)

i.e, $72n^4 - 24n^3 > 162\sqrt{2}n^2 + (24\sqrt{10} - 54\sqrt{2})n$.

(ii) MDI(OX_n) >^{m} EΔ(OX_n)

$$i.e, 72n^4 - 24n^3 > \frac{6n}{4\sqrt{10}} + \frac{9n^2 - 3n}{18\sqrt{2}}$$

D. Minimum Degree Index for Hexagonal Network:

In graph theory, a hexagonal network is a network that is modeled by a planar graph and is based on a triangular plane tessellation. This means that the network is made up of equilateral triangles, with each node having up to six neighbors. The total count of edges and vertices in a hexagonal network are (9n2 − 15n + 6) and (3n2 − 3n + 1) correspondingly, there exists five types of edges based on the vertices and degrees of each edge as follows.

$$E_{3,4} = \{u_iv_i\epsilon E(G)\ |d_G(u_i) = 3, d_G(v_i) = 4\}, |E_{3,3}| = 12.$$

$$E_{3,6} = \{u_iv_i\epsilon E(G)\ |d_G(u_i) = 3, d_G(v_i) = 6\}, |E_{3,6}| = 6.$$

$$E_{4,4} = \{u_iv_i\epsilon E(G)\ |d_G(u_i) = d_G(v_i) = 6\}, |E_{6,6}| = 16n - 18.$$

$$E_{4,6} = \{u_iv_i\epsilon E(G)\ |d_G(u_i) = 3, d_G(v_i) = 6\}, |E_{3,6}| = 12n - 24.$$

$$E_{6,6} = \{u_i v_i \epsilon E(G) \,|d_G(u_i) = d_G(v_i) = 6\}, |E_{6,6}| = 9n^2 - 33n + 30.$$

Theorem 4.4. MDI for Hexagonal Network HX_n Then,

$$MDI(HX_n) = 54n^4 - 126n^3 + 66n^2.$$

Proof. By definition we have,

$$\mathrm{MDI}(HX_n) = n^2 \left(\sum_{u,v \in E} min[d(u), d(v)] \right) \qquad (5)$$

$$= 54n^4 - 126n^3 + 66n^2.$$

Also we have the following results for HX_n

$$\mathrm{MDI}\ (HX_n) > \mathrm{E}\Delta\ (HX_n)$$

i.e.,$54n^4 - 126n^3 + 66n^2 > 288\sqrt{2}n^2 + (144\sqrt{5} - 1008\sqrt{2})n + 30\sqrt{17} - 252\sqrt{5} + 816\sqrt{2}$.

(ii) $\mathrm{MDI}(HX_n) >^m \mathrm{E}\Delta(HX_n)$

i.e.,$54n^4 - 126n^3 + 66n^2 > \frac{12}{3\sqrt{5}} + \frac{6}{5\sqrt{17}} + \frac{6n-18}{8\sqrt{2}} + \frac{12n-24}{12\sqrt{5}} + \frac{9n^2-33n+30}{32\sqrt{2}}$.

V. CONCLUSION

Using the definition of Minimum Degree Index we have computed Minimum Degree Index for few standard graphs. We have also obtained Minimum Degree Index for Silicate, Honeycomb, Oxide and Hexagonal networks and compared it with elliptic delta index and modified elliptic delta index. We have observed that MDI has a greater value compared to elliptic delta and modified elliptic delta indices. Further these values can be utilized to derive the nature of the chemical structures through MDI values.

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