

Application of Switching Functions on Inverted Pendulum System

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Abstract—In several investigations on control systems, sliding mode control has been identified as a beneficial instrument. The system is stable due to SMC's tolerance to instabilities and model heterogeneity. Furthermore, the chattering phenomena hinders system performance and may possibly lead to disruption. This paper deals with the application of different switching functions on inverted pendulum system. Further the comparison between them has also been done. Simulation results are given to demonstrate the effectiveness of the proposed approaches.

Index Terms— inverted pendulum, sliding mode control (smc), dynamical object, non-linear control, saturation function, signum function, Hyperbolic function.

I. INTRODUCTION

In industrial applications, inverted-pendulum systems always present a number of issues, such as variable nonlinear behaviours under different operating circumstances. Extrinsic interferences and physical restraints some variables.[1] Consequently, the objective of real-time stabilising controlling the tracking of a highly nonlinear unstable inverted pendulum system has been a difficulty for the current control community. In recent years, the control difficulties of inverted-pendulum systems have been thoroughly researched due to the challenging requirement for rapid and precise performance[2]. The mechanical structure of these systems can be categorised into three basic kinds, as described in the literature. One is a pendulum with one degree of freedom (dof) set on a base that rotates around the vertical axis or is controlled by a linear slide. The second is a double[3] or triple inverted pendulum[4] with many dofs that is driven by a cart. A second example is a two-dof pendulum mounted on a robot with decoupled or weakly coupled linkages. Earlier research include controlling methods like pd,pid,mpc on inverted pendulum[5].

The inverted pendulum device is recognized basic experimental in electronic control control theory and a common physical prototype in control theory education and research. The system's stability control is remarkably equivalent to those of walking robots, aircraft, and elevator hook devices[6]. As a controlled device, the inverted pendulum system is inexpensive and seems to have a basic design. Like a control mechanism, an inverted pendulum system has a basic structure and a cheap cost. Combining mechanics, mathematics, and electrical, In the system of inverted pendulums, the control device has a wide range of applications. The inverted pendulum, as a controlled object, is a greater, volatile, multivariate, non - linear, strong-coupling, and rapid system.[7] Only if suitable control mechanisms are established can it be stable. The inverted pendulum would be both broad and representative of stability control issues. Consequently, it has substantial theoretical relevance and real applications significance for pendulum exploration.

II. INVERTED PENDULUM MODEL

There are issues in IP system modelling due to its volatility. But after disregarding certain less significant aspects, the IP system resembles a conventional stiff body in motion.

The development of the pendulum framework contains rotatory and translator motion.

The equations are given as [8]-

$$\ddot{\theta} = \frac{m(m+M)gl}{(M+m)I+Mml^2}\theta - \frac{ml}{(M+m)I+Mml^2}(z + d(t)) \quad (1)$$

$$\ddot{x} = -\frac{m^2gl^3}{(M+m)I+Mml^2}\theta + \frac{1+ml^2}{(M+m)I+Mml^2}(z + d(t)) \quad (2)$$

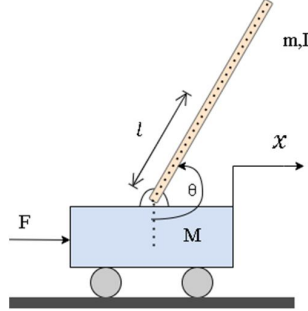


Fig 1 FBD of Inverted Pendulum

III. SLIDING MODE CONTROL

SMC is a kind of variable structural control scheme. During the control process, the control law is changed based on a combination of criteria that have been contingent on the state of the system. It delivers amazing results for structures with matched uncertainties.

Further, it possesses disturbance rejection even in the existence of model errors, although at the price of controlling chattering and a period wherein the dynamic analysis is subject to uncertainty. design of a robust switching surface and development of a control law to force the systems stages onto the selected surface within one finite amount of time.[9]

With input reference, the sliding surface is created such that the state traceability corresponds to zero. Therefore, once the system reaches the sliding mode, the system trajectories must drop down this surface. The swapping circuit grants the non-linear control action to confine system pathways to the sliding surface. Defining then $t_1 = \theta, t_2 = x$ then $w_1 = \theta$,

$$w_2 = \dot{x}. \text{ Define } f_1(t, w) = \frac{m(m+M)gl}{(M+m)I+Mml^2}t_1$$

$$g_1(w_2) = \frac{ml}{(M+m)I+Mml^2}, \quad f_2(t, w) = -\frac{m^2gl^2}{(M+m)I+Mml^2}t_1, \text{ and}$$

$$g_2(w_2) = \frac{I+ml^2}{(M+m)I+Mml^2}$$

Then,

$$\frac{g_1(w_2)}{g_2(w_2)} = \frac{\frac{ml}{(M+m)I+Mml^2}}{\frac{I+ml^2}{(M+m)I+Mml^2}} = -\frac{ml}{I+ml^2} \quad \#(3)$$

By using decoupling algorithm,

$$\gamma_1 = t_1 - \int_0^{t_2} \frac{g_1(s)}{g_2(s)} ds = t_1 + \frac{ml}{I+ml^2}t_2 \quad \#(4)$$

$$\gamma_2 = t_1 - \frac{g_1(t_1)}{g_1(t_2)}w_2 = w_1 + \frac{ml}{I+ml^2}t_2 \quad \#(5)$$

$$\delta_2 = w_2, \delta_1 = t_2$$

The system under stable point when both pendulum angle and cart position both comes at rest position, means $\theta = 0, x = 0, \dot{\theta} = 0, \dot{x} = 0$ Then $\gamma_1 = 0, \gamma_2 = 0, \delta_1 = 0, \delta_2 = 0$

$$\dot{\gamma}_2 = \left(\frac{m(m+M)gl}{(M+m)l + Mml^2} - \frac{ml}{I + ml^2} \frac{m^2 gl^2}{(M+m)l + Mml^2} \right) t_1 \quad \#(6)$$

$$\dot{\delta}_2 = -\frac{m^2 gl^2}{(M+m)I + Mml^2} w_1 + \frac{I + ml^2}{(M+m)l + Mml^2} (u + d(t)) \quad \#(7)$$

Let

$$T_1 = \left(\frac{m(m+M)gl}{(M+m)I + Mml^2} - \frac{ml}{I + ml^2} \frac{m^2 gl^2}{(M+m)I + Mml^2} \right) \quad \#(8)$$

$$T_2 = -\frac{m^2 gl^2}{(M+m)I + Mml^2} \quad \#(9)$$

$$T_3 = \frac{I + ml^2}{(M+m)I + Mml^2} \quad \#(10)$$

Then Equation becomes [10]

$$\dot{\gamma}_1 = \gamma_2 \quad \#(11)$$

$$\dot{\gamma}_2 = T_1 t_1 \quad \#(12)$$

$$\dot{\delta}_1 = \delta_2 \quad \#(13)$$

$$\dot{\delta}_2 = -T_2 w_1 + T_3 (z + d(t)) \quad \#(14)$$

IV. CONTROLLER DESIGN

Let $\mu_1 = \delta_2, \mu_2 = [\gamma_1 \quad \gamma_2 \quad \delta_1]^T$

Design the sliding mode controller as,

$$\sigma = \mu_1 - C\mu_2$$

Where $C = [c_1 \quad c_2 \quad c_3]$

Then we have $\dot{\sigma} = \dot{\mu}_1 - C\dot{\mu}_2$

$$\dot{\sigma} = T_2 \gamma_1 + T_2 T_4 \delta_1 + T_3 (u + d(t)) - C\dot{\mu}_2$$

Design the sliding mode controller as

$$z = \frac{1}{T_3} (-T_2 \gamma_1 - T_2 T_4 \delta_1 + C\dot{\mu}_2 - h \operatorname{sgn}(\sigma)) \quad \#(15)$$

Where $h \geq |d(t)|$.

V. SIMULATIONS

The following parameters were chosen for simulation

$M=1; m=0.010; L=0.50$ Initial States as $\theta(0) = -10^\circ, \dot{\theta}(0) = 0, x(0) = 0.20, \dot{x}(0) = 0$

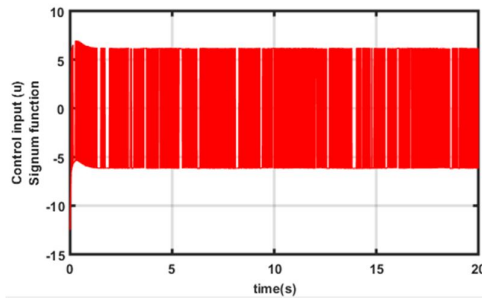


Fig 2 Control input with Sign function

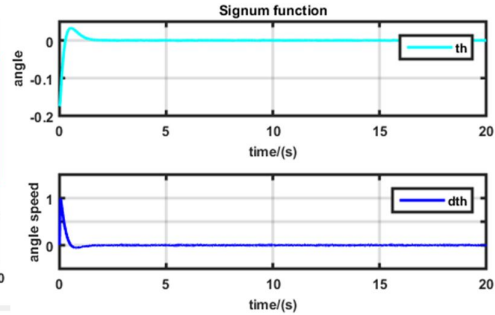


Fig 3 Angle plots using Sign function

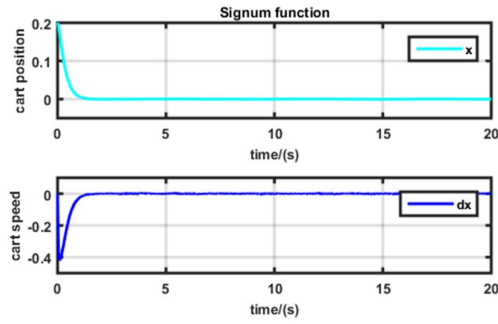


Fig 4 Cart plots using Sign function

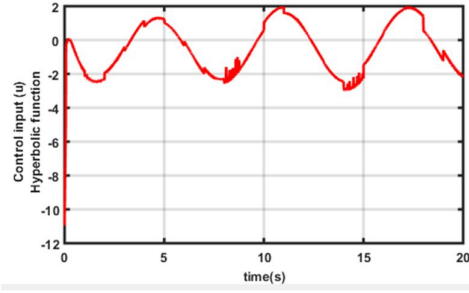


Fig 5 Control input with tanh function

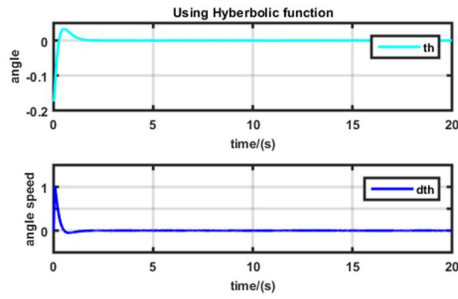


Fig 6 Angle plots using tanh function

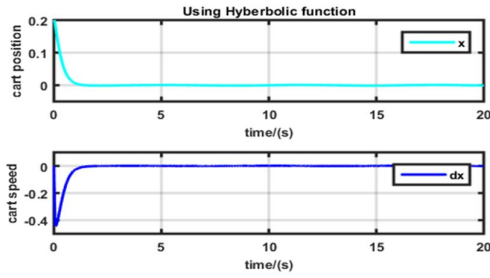


Fig 7 Cart plots using tanh function

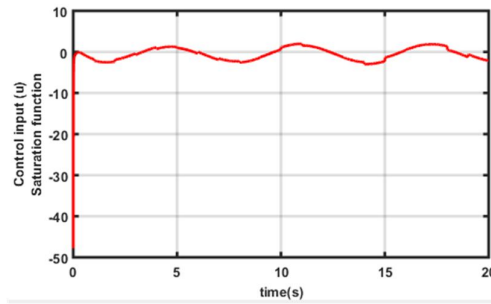


Fig 8 Control input using Sat function

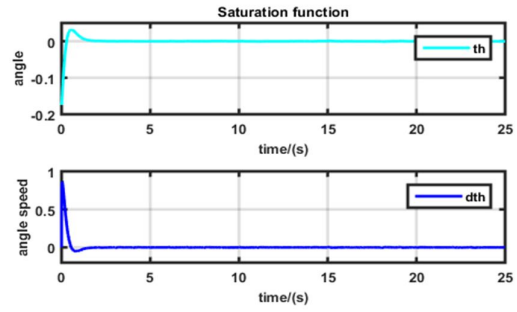


Fig 9 Angle plots using Sat function

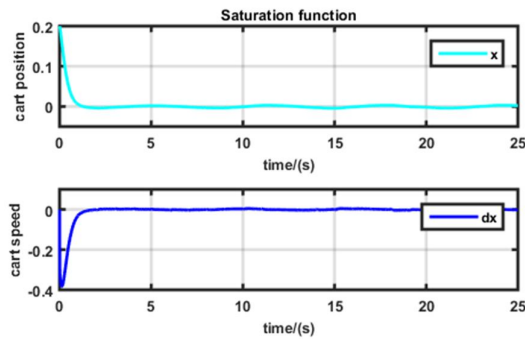


Fig 10 Cart plots using Sat function

VI. RESULT ANALYSIS

The table I shows a comparison of control input using signum, saturation and hyperbolic function. We can see a decent hike in rise time whereas the overshoot, rms, peak to peak values are falling. Thus this subjective analysis shows that sliding mode control with saturation function gives good results as comparison to signum function

and hyperbolic function. Also the major problem coming in signum function and hyperbolic i.e. chattering has also been reduced.

TABLE I.

Parameters	Control input using Signum function	Control input using Sat function	Control input using tanh function
Amplitude	1.227e+01	4.531e+01	8.545
Slew Rate	24.277	124.921	94.120
Rise time	497.942 μ s	5.548 ms	54.72ms
Overshoot	0.611%	0.367%	0.482%
RMS	6.145	4.097	7.341

VII. CONCLUSION

Numerous researchers have assessed the inverted pendulum as a prominent benchmark for designing and implementing different controllers. This is because of its complex nature. Therefore, a sliding mode controller has been designed and analysed for the inverted pendulum system in this paper. First, we generated its dynamical mathematical model. Then, we devised a sliding mode controller and implemented several switching functions, including signum, saturation, and hyperbolic functions, examining their responses to angle, cart, and control input. Also, we have introduced disturbances and noise. The results indicate that the saturation function is superior to the other two switching functions due to its peculiarity to eliminate chattering and reduce error and overshoot.

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