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Analysis of the Cumulative Distribution of Peak-to-Average Power Ratio for OFDM Signals

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***Abstract*— When combined coherently, the many separately modulated SCs that make up an OFDM signal can result in a high peak-to-average power (PAP) ratio. The power of a sine wave with an amplitude equal to the maximum envelope value is known as the peak power. As a result, the PAP ratio of an unmodulated carrier is 0 dB.**
The Crest factor, which is calculated by dividing the maximum signal value by the RMS signal value, is an alternate indicator of a signal's envelope fluctuation. The Crest factor for an unmodulated carrier is 3 dB. As long as the center frequency is high relative to the signal bandwidth, this 3-dB discrepancy between the PAP ratio and Crest factor also applies to other signals.

***Index Terms*— SubCarriers (SCs), Peak-to-Average Power (PAP), Orthogonal Frequency Division Multiplexing (OFDM), Forward Error Correction (FEC), Quadrature Phase Shift Keying (QPSK), Cumulative Distribution Function (CDF), Signal-to-Noise Ratio (SNR), Coded OFDM (COFDM), Parallel-to-Serial (P/S).**

I. INTRODUCTION

Reduced RF power amplifier efficiency and more complex analog-to-digital and digital-to-analog converters are two disadvantages of a high PAP ratio.
Many strategies have been developed to reduce the PAP ratio; these strategies can be roughly divided into three categories. First, there are signal distortion techniques that reduce the peak amplitudes by merely nonlinearly corrupting the OFDM signal at or close to the peaks. Distortion techniques include peak windowing, peak cancellation, and clipping.
Second, certain coding techniques remove OFDM symbols with large PAP ratios by using a special FEC code set. After scrambling each OFDM symbol using multiple scrambling sequences, the third technique selects the scrambling sequence that yields the lowest PAP ratio.
This paper covers all of these techniques, but first it looks at the PAP ratio distribution function. This will demonstrate why PAP reduction techniques can be quite effective and provide additional insight into the PAP problem.

II. DISTRIBUTION OF THE PAP RATIO

The complex baseband signal for a single OFDM symbol with N SCs can be expressed as

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$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n \, exp(j\omega_n t) \qquad (1)$$

The modulating symbols in this case are a_n. For example, in QPSK, $a_n \in \{-1, 1, j, -j\}$. The real and imaginary values of $x(t)$ become Gaussian distributed for high values of *N*, with a mean of zero and a variance of $1/2$, according to the central limit theorem. As a result, the OFDM signal's amplitude has a Rayleigh distribution, and its power distribution turns into a central chi-square distribution with two degrees of freedom, zero mean, and a cumulative distribution given by

$$F(z) = 1 - e^{-z} \qquad (2)$$

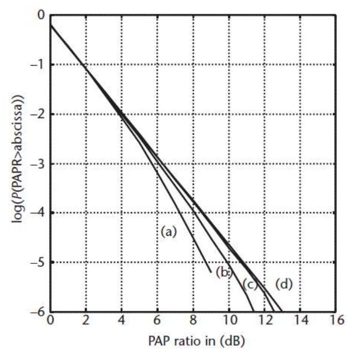

Fig 1. PAP distribution of an OFDM signal with (a) 12, (b) 24, (c) 48, and (d) an infinite number of SCs (pure Gaussian noise). A total of 12 million simulated samples were created using four times oversampling.

The likelihood that the PAP ratio may surpass a particular number is displayed in Figure 1. Until the PAP value is within a few decibels of the maximum PAP level of 10logN, where N is the number of SCs, we can observe that the curves for different numbers of SCs are near to the Gaussian distribution displayed in the data.

We now wish to calculate the peak power per OFDM symbol's CDF. The likelihood that the PAP ratio is below a given threshold level can be expressed as follows, assuming the samples are mutually uncorrelated, which is true for no oversampling:

$$P(PAPR \leq z) = F(z)^N = (1 - \exp(-z))^N \qquad (3)$$

In Figure 2, this theoretical derivation is displayed versus simulated values for various *N* values.

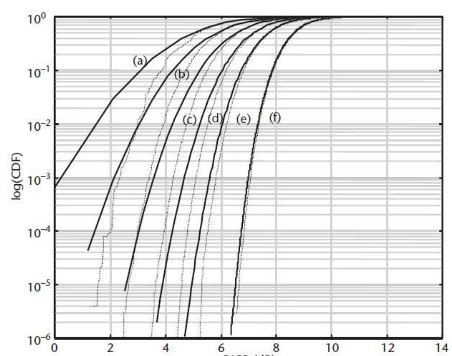

Fig 2. PAP distribution for (a) 16, (b) 32, (c) 64, (d) 128, (e) 256, and (f) 1,024 SCs without oversampling. Simulated lines are represented by dots.

When oversampling is used, the premise that the samples should be mutually uncorrelated that was used in deriving (3) is no longer valid. We suggest an approximation by assuming that the distribution for αN SCs and oversampling can be approximated by the distribution for αN SCs without oversampling, with $\alpha > 1$, since it appears to be fairly difficult to find an accurate solution for the peak power distribution. Therefore, a specific number of additional independent samples are added to simulate the effect of oversampling.

Next, the PAP ratio distribution is provided by

$$P(PAPR \leq z) = F(z)^N = (1 - \exp(-z))^{\alpha - N} \qquad (4)$$

The PAP distribution for various carrier counts for $\alpha = 2.8$ is shown in Figure 3. Simulated curves are represented by the dotted lines. Figure 3 shows that, for $N > 64$, Eq. 4 is rather true. However, (3) is actually more accurate for big values of the CDF near one (> 0.5).

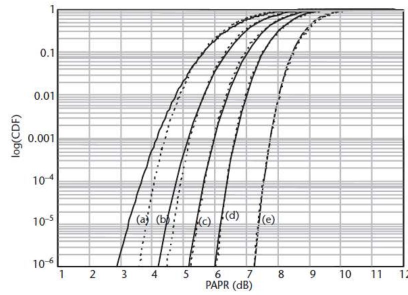

Figure 3 CDF of the PAP ratio (PAPR) for the following SC numbers: (a) 32, (b) 64, (c) 128, (d) 256, and (e) 1,024. Dotted lines are simulated, while solid lines are computed.

Given that good coding rates are achievable for a PAP ratio of about 4 dB, Figure 3 suggests that a coding approach to lower the PAP ratio may be a feasible choice. For example, approximately 10^(-6) of all feasible QPSK symbols for 64 SCs have a PAP ratio of less than 4.2 dB. This indicates that if only the symbols with a low PAP ratio were transmitted, only 20 of the 128 bits would be lost. Finding a coding scheme with a reasonable coding rate (≥1/2) that generates exclusively these low PAP ratio symbols and has reasonable error-correcting qualities is the primary challenge with this method, though.

An alternative method for solving the PAP problem is to take use of the fact that big PAP ratios are rare and can be eliminated at the expense of a little amount of self-interference. The current challenge is to minimize the spectrum pollution caused by this self-interference. One PAP reduction method that produces self-interference is clipping. Two other methods with superior spectral qualities to clipping are discussed in the following sections.

III. Clipping and Peak Windowing

Clipping the signal to limit the peak amplitude to a desired maximum level is the most straightforward method of lowering the PAP ratio. Clipping is undoubtedly the easiest approach, but it has certain drawbacks. First, a type of self-interference that lowers the BER is introduced by altering the amplitude of the OFDM signal. Second, the amount of out-of-band radiation is greatly increased by the OFDM signal's nonlinear distortion. By considering the clipping operation as a multiplication of the OFDM signal by a rectangle window function, which equals one if the OFDM amplitude is below a threshold and less than one if the amplitude has to be clipped, the latter impact may be clearly understood. The input OFDM spectrum convolved with the window function's spectrum yields the clipped OFDM signal's spectrum. The rectangular window function's spectrum, which is the wider of the two, primarily determines the out-of-band spectral characteristics. The roll off of this spectrum is extremely slow and inversely proportional to the frequency.

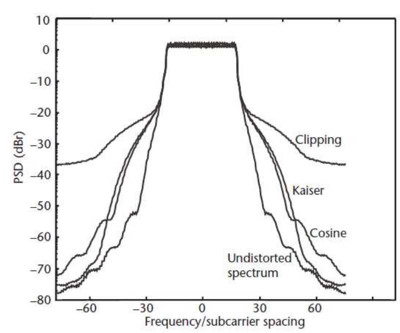

Figure 4 Frequency spectrum of an OFDM signal with 32 SCs with clipping and peak windowing at a threshold level of 3 dB above the RMS amplitude.

The distinction between windowing and clipping the signal is seen in Figure 4. Figure 5 illustrates how the spectral distortion can be reduced by widening the window.

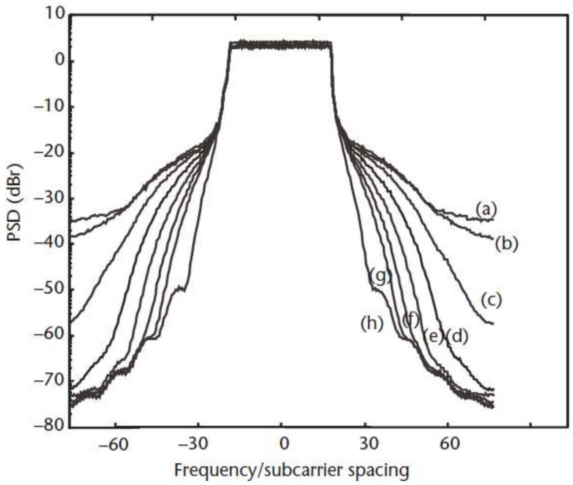

Figure 5 An OFDM signal's frequency spectrum with 32 SCs and peak windowing at a threshold of three decibels above the RMS amplitude. The window length is (a) 3, (b) 5, (c) 7, (d) 9, (e) 11, (f) 13, and (g) 15 samples, whereas the symbol length is 128 samples (four times oversampled). The optimal OFDM spectrum is shown by curve (h).

Using a rate 1/2 convolutional code with constraint length 7, Figure 6 displays packet-error ratio (PER) curves with and without clipping. 48 SCs with 16-QAM are used in the simulated OFDM signal. The graphs show that nonlinear distortion only slightly affects the PER; when the PAP ratio is reduced to 6 dB, the SNR loss is roughly 0.25 dB. Because peak windowing distorts a greater portion of the signal than clipping for the same PAP ratio, the results are somewhat worse when peak windowing is used.

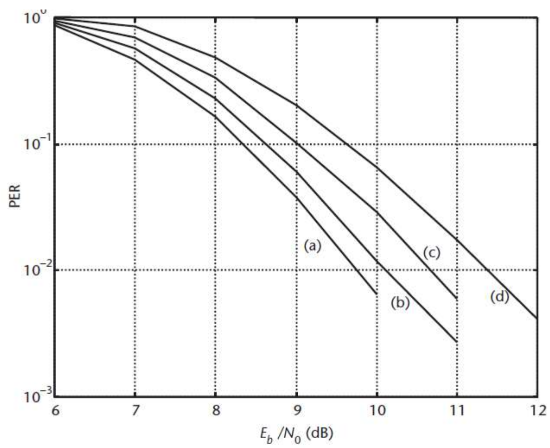

Figure 6 PER against *Eb/N*0 for AWGN 64-byte packets. The OFDM signal is clipped to a PAP ratio of (a) 16 (no distortion), (b) 6, (c) 5, and (d) 4 dB.

IV. Coding and Scrambling

One drawback of distortion methods is that symbols with a high PAP ratio deteriorate more and are therefore more prone to mistakes. FEC coding can be used across several OFDM symbols to lessen this effect. This allows the surrounding symbols to correct errors produced by symbols with a large deterioration. The mistake probability in a COFDM system depends on the power of several consecutive symbols rather than the strength of individual signals. Assume, for instance, that if more than four out of ten symbols have a PAP ratio greater than 10 dB, the FEC code generates an error. Additionally, consider that there is a 10^{-3} chance of having a PAP ratio greater than 10 dB. Next, the peak cancellation technique's error probability is

$$1-\sum_{i=0}^{3}\binom{10}{1}\,(10^{-3})^{i}(1-10^{-3})^{10-i}\;\cong\;2.10^{-10}$$

less than the 10^{-3} in the absence of FEC coding.

For real-time circuit-switched traffic, like speech, such a low symbol-error probability might be sufficient, but packet data might still experience issues. An excessive number of large PAP ratio symbols in a packet increases the likelihood of a mistake. As demonstrated above, such packets are extremely rare, but when they do occur, they might never be successful because each retransmission of the packet has the same high error probability.

V. PEAK CANCELLATION

Reducing the amplitude of samples whose power beyond a particular threshold is the fundamental component of all distortion algorithms. A nonlinear distortion of the OFDM signal produced a certain quantity of out-of-band radiation in the case of clipping and peak windowing. By using a linear peak-cancellation technique, which involves subtracting a time-shifted and scaled reference function from the signal so that each subtracted reference function lowers the peak power of at least one signal sample, this unwanted impact can be prevented. It is possible to ensure that the peak power reduction won't result in any out-of-band interference by choosing a suitable reference function with roughly the same bandwidth as the transmitted signal.

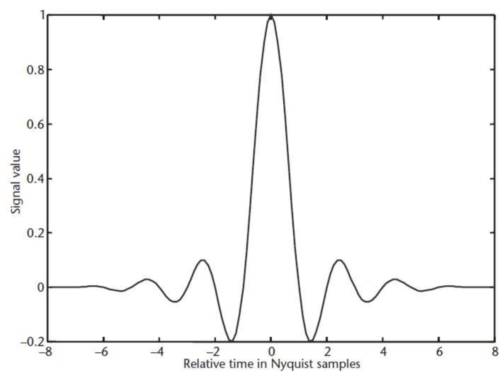

Figure 7 A raised cosine window is used to window the sinc reference function.

An example of a reference function is shown in Figure 7, which is produced by multiplying a raised cosine window by a sinc function. It is guaranteed that the reference function has the same bandwidth as the standard OFDM signals if the windowing function is the same as that used to window the OFDM symbols. Peak cancellation will therefore not deteriorate the characteristics of the out-of-band spectrum. A trade-off between reduced complexity in peak-cancellation computations and a slight increase in out-of-band power can be achieved by narrowing the reference signal window.

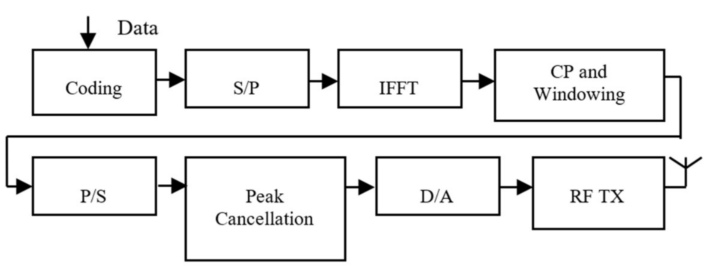

Figure 8 OFDM transmitter with peak cancellation

After the digital OFDM symbols are generated, peak cancellation can be done digitally. It includes a comparator to determine whether the peak power surpasses a threshold, a peak power (or peak amplitude) detector, and a scaling of the peak and surrounding samples. The block diagram of an OFDM transmitter with peak cancellation is displayed in Figure 8. Initially, incoming data is coded and transformed into blocks of N complex signal samples from a serial bit stream. An IFFT is carried out on each of these blocks.

The symbol size is then increased to N + NG samples by adding a cyclic prefix. The peak cancellation technique is used to lower the PAP ratio following parallel-to-serial (P/S) conversion. Peak cancellation can also be done right after the IFFT and prior to windowing and the cyclic prefix. There are no other differences from a typical OFDM transmitter, other from the peak cancellation block. Any conventional OFDM receiver can be utilized because there is absolutely no change for the receiver.

VI. CONCLUSION

These peaks can be eliminated at the expense of a small bit of self-interference. The current challenge is to minimize the spectrum pollution caused by this self-interference. Because peak windowing distorts a greater portion of the signal than clipping for the same PAP ratio, the results are somewhat worse when peak windowing is used. A solid-state power amplifier's AM/PM conversion is negligible enough to be disregarded. Compared to clipping or peak windowing, peak cancellation appears to be a quite distinct strategy. Peak cancellation may be demonstrated to be nearly equal to clipping and then filtering.

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