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Exact Pendant Edge Domination in Graphs

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***Abstract*— Let G = (V, E) be a connected graph. A pendant edge dominating set F of G is called a exact pendant edge dominating set (expedset), if the edge induced subgraph $\langle N(F)\rangle$contains at least one pendant edge. The minimum cardinality of a expedset of G is called the exat pendant edge domination number of G and is denoted by $\gamma'_{epe}(G)$. In this paper We initiate a study of this parameter.**

***Keywords*— Exact domination, Exact edge domination, Exact pendant edge domination.**
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I. INTRODUCTION

By the graph G = (V, E) we mean a simple, connected, finite and undirected graph. The order and size of graph (G) are denoted by n and m respectively. For graph terminology, we refer to [2].
Let G = (V, E) be a graph and $v \in V.$ The open neighborhood and closed neighborhood of v are denoted by $N(v)$ and $N[v] = N(v) \cup \{v\}$ respectively. If $S \subseteq$ V, then $N(S) = \bigcup_{v\in S} N(v)$and $N[S] = N(S) \cup S.$ For any $e \in E.$ The open neighborhood and closed neighborhood of e are denoted by $N(e)$ and $N[e] = N(e) \cup \{e\}$ respectively. If $X \subseteq$ E, then $N(X) = \bigcup_{e\in X} N(e)$and $N[X] = N(X) \cup X.$ If $X \subseteq$ E and $e_1 \in X$, the degree of an edge $e = uv\ of\ G$ is defined by $dege = degu + degv - 2.$ $\delta'(G), (\Delta'(G))$ is the minimum (maximum) degree among the edges of G. Let $X \subseteq$ E, a graph $G - X$ is obtained from the graph G by removing the edges of X. Let H be a subgraph of G and $e \in E$ denotes the distance from e to H. For a graph G, its line graph denoted by L(G) is defined to be a graph whose vertex set will be the edge set of G and two vertices in L(G) are adjacent if corresponding edges are adjacent in G. The dominating set of G and its line graph L(G) are related in the sense that vertices dominated in L(G) represents edges dominated in G [4]. We recall the following result, which is require for our study.
A coloring of a graph G is an assignment of colours to the edges of G such that no two adjacent edges receive the same colour. The minimum integer k for which a graph G is k-colorable is called chromatic number of G and is denoted by χ(G).
Definition1.1.[3] A subset S of V(G) is a dominating set of G if each vertex u$\in$ V−S is adjacent to at least one vertex in S. The least cardinality of a dominating set in G is called the domination number of G and is usually denoted by γ(G).
Definition1.2. [5]A set X $\subseteq$ E is an edge dominatingset if every edge E $-$ X is adjacent to some edge in X. The minimum cardinality of edge dominating set is called the edge domination number, denoted by γ'(G).

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Definition1.3. [6] An edge dominatingset F of a graph G is called pendant edge dominating set , if the induced subgraph ⟨F⟩ contains atleast one pendant edge. The minimum cardinality of a pendant edge dominating set of G is called pendant edge domination number, denoted by $\gamma'_{pe}(G)$.

Definition1.4.[1] Let G = (V, E) be a connected graph. Let $G = (V, E)$ be any connected graph. Let $F \subseteq E$. The set F is said to be an exact edge dominating set , if $|N(e_i) \cap F| = 1$ and $|N(e_j) \cap F| \leq 1$ for every $e_i \in E(G) - F$ and $e_j \in F$. An exact edge dominating set is denoted as ExED set. The exact edge domination number by $\gamma'_e(G)$ of a graph equals the cardinality of a minimum exact edge dominating set

Definition1.5. An edge dominatingset F of a connected graph G is called exact pendant edge dominating set, if the induced subgraph⟨N(F)⟩contains atleast one pendant edge. The minimum cardinality of exact pendant edge dominating set of G is called exact pendant edge domination number, denoted by $\gamma'_{epe}(G)$.

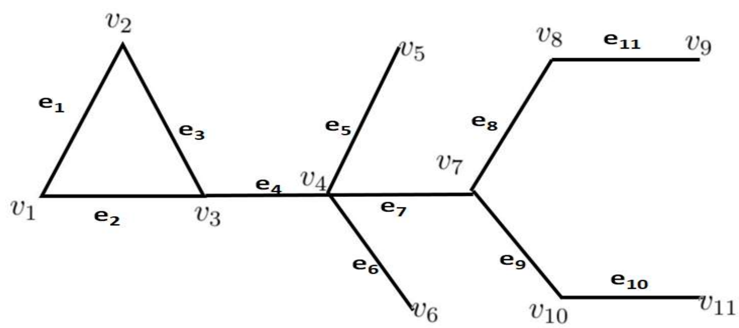

Fig 1

The set $F = \{e_1, e_5, e_{10}, e_{11}\}$ is an exact pendant edge dominating set. The set $F = \{e_1, e_7, e_{10}, e_{11}\}$ is not an exact pendant edge dominating set because $e_8 \in E - F$ and $|N(e_8) \cap F| \neq 1$.

Observation 1.1: It is noticed that every exact pendant equitable dominating set is a dominating set but the converse is not true. That is, every dominating set need not be an exact pendant equitable dominating set.

II. Main Results

Definition 2.1. A pendant edge dominating set F of connected graph G is called a exact pendant edge dominating set (exped-set) if the edge induced subgraph ⟨N(F)⟩contains atleast one pendant edge. The minimum cardinality of a exped-set of G is called the exact pendant edge domination number of G and is denoted by $\gamma'_{epe}(G)$.

Remark

clearly $\gamma'_{epe} \geq \gamma'$. Further if F is a exact edge dominating set with $|F| > 1$. Then $N(F) = E$ and hence $\gamma'_{epe} \geq \gamma'_e(G)$

Theorem 2.2. For any graph G , $\gamma'(G) \leq \gamma'_{epe}(G) \leq 2\gamma'(G)$. Further given two positive integers a and b with $a \leq b \leq 2a$. There exists a graph G with $\gamma'(G) = a \; and \; \gamma'_{ncpe}(G) = b$.

Proof. Let a and b be positive integers with $a \leq b \leq 2a$. Let $b = a + k, 0 \leq k \leq a. \; If \; k = 0, \; let \; G = k_a o \; k_1$. Suppose $1 \leq k \leq a$. Consider the galaxy of stars $G_1, G_2, G_3, \ldots G_a$with $|V(G_i)| \geq 3, \; 1 \leq i \leq a$. Join the maximum of degree vetices of G_i and G_{i+1} By an edge e_i.

$1 \leq i \leq a$. Let H be the graph obtained from the above graph by subdividing exactly once the edges e_i. $1 \leq i \leq a - 1$. clearly $\gamma'(H) = \gamma'_{epe}(H) = a$. Let G be the graph obtained from H by subdividing an edge $G_i$exactly once where$1 \leq i \leq k$. Then $\gamma'(G) = a$ and $\gamma'_{epe}(G) = a + k = b$.

Theorem 2.3. Let $G = P_n$ be a path with n vertices then

$$\gamma'_{epe}(G) = \begin{cases} \frac{n}{3} + 1, & if \; n \equiv 0 \; (mod3) \\ \left\lceil \frac{n}{3} \right\rceil, & if \; n \equiv 1 \; (mod3) \\ \left\lceil \frac{n}{3} \right\rceil + 1, & if \; n \equiv 2 (mod3) \end{cases}$$

Proof: Let $G \cong P_n$ be a path and let $E(P_n) = \{e_1, e_2, e_3, \ldots, e_n\}$.
We consider the following possible cases.
Case (i): Suppose $n \equiv 0(mod3)$ i.e $n = 3k$ for some integer $k > 0$ then the set
$S = \{e_1, e_{3i-1}/1 \leq i \leq k\}$is an exact pendant edge dominating set of G. Thus, $\gamma'_{epe}(P_n) = \frac{n}{3} + 1$.
Case (ii): Suppose $n \equiv 1(mod3)$ then it is easy to check the exact pendant edge dominating set. The exact pendant edge set itself is an exact pendant edge dominating set in G. Therefore, $\gamma'_{epe}(P_n) = \left\lceil\frac{n}{3}\right\rceil$.
Case (iii): Proof of this case is similar to case (i).
Corollary 2.4. For any non-trivial path p_n,

- $\gamma'_{epe}(p_n) = \gamma'(p_n)$ iff n = 5.
- $\gamma'_{epe}(p_n) = \gamma'_e(p_n)$ iff $n \equiv 2(mod3)$

Proof: Since $\gamma'(p_n) = \left\lfloor\frac{n-1}{3}\right\rfloor$ and $\gamma'_e(p_n) = \left\lfloor\frac{n}{3}\right\rfloor + 1,\ if\ n \equiv 2(mod3)$
the corollary follows.
Theorem 2.5. For the cycle C_n , $n > 3$

$$\gamma'_{ncpe}(C_n) = \left\lceil\frac{n}{3}\right\rceil,\quad iff\ \ n \equiv 1\ (mod3)$$

Proof: Let $G \cong C_n$ be a cycle and let $E(P_n) = \{e_1, e_2, e_3, \ldots, e_n\}$.
Suppose $n \equiv 1(mod3)$ i.e $n = 3k + 1$ for some integer $k > 0$ then the set
$S = \{e_1, e_{3i-1}/1 \leq i \leq k\}$is an exact pendant edge dominating set of G. Thus, $\gamma'_{epe}(c_n)$= $\left\lceil\frac{n}{3}\right\rceil$.
Theorem 2.6: Let G be a complete bipartite graph of order $2n$, then $\gamma'_{epe}(G) = 2$.
Proof: Let G be a complete bipartite graph and $\{e_1, e_2, e_3, \ldots, e_{2n}\}$ are edges of G. Let us choose the set $F = \{e_1, e_{n+1}\}$ where e_1 and e_{n+1} are two adjacent edges in G. Then the set F will be a minimal exact pendant edge dominating set in G. Hence, $\gamma'_{epe}(G) = 2$.
Theorem 2.7: Let G be a Lollipop graph then $\gamma'_{epe}(G) = \left\lfloor\frac{n}{3}\right\rfloor + 2$.
Proof: Let G be a Lollipop graph of the form $L_{m,n}$where $m = 3$ and $n = 1,2,3, \ldots$.
We have possible two cases.
Case (i): For $m = 3$ and $n = 3k, 3k + 1$ where $k > 0$. Fix the edge e_1 and the set $S = \{e_1, v_{3i-1}\}$for $i = 1,2,3, \ldots$,will be the minimum exact pendant edge dominating set.
Case (ii): For $m = 3$ and $n = 3k + 2$ where $k > 0$. The set $S = \{e_2, v_{3i}\}$
for $i = 1,2,3, \ldots$, will be the minimum exact pendant edge dominating set.
Theorem 2.8: Let G be a Pan graph then $\gamma'_{epe}(G) = 3 + \left\lfloor\frac{n-5}{3}\right\rfloor$.
Proof: Consider a Pan graph with n vertices and the edge set contain $F = \{e_1, e_2, e_3, \ldots, e_n\}$ where $e_1$is an edge of degree two incident to a vertex of degree one. We consider the following cases.
Case (i): Suppose $n \equiv 0(mod3)$i.e$n = 3k$ for some integer $k > 0$. Fix the edge of degree two which is incident to a vertex of degree one and its adjacent edge with degree two in the cycle. Remove neighbourhood vertices of degree three then the graph obtained after removing the edge and the corresponding vertices is isomorphic to P_{n-3}.
Then, the exact pendant edge domination number of the Pan graph is $\gamma'_{epe}(G) = 3 + \gamma'(n-3) = 3 + \left\lfloor\frac{n-5}{3}\right\rfloor$.
Case (ii): Suppose $n \equiv 1(mod3)$. Fix the edges$e_n$and e_{n-1}in the cycle and the vertex e_n is connected to the singleton set in the bridge. Remove the neighboring edges of $e_n$and e_{n-1} from the pan graph then the graph obtained after removing the edges and corresponding vertices is isomorphic to P_{n-3} .
Then, the exact pendant edge dominating number of the Pan graph is $\gamma'_{epe}(G) = 3 + \gamma'(n-3) = 3 + \left\lfloor\frac{n-5}{3}\right\rfloor$.
Remark: For $n \equiv 1(mod3)$ pan graph has no exact pendant edge dominating number.
Bounds for Exact Pendant Edge domination number $\gamma'_{epe}(G)$
Theorem 3.1: Let S be a minimum exact pendant edge dominating set in G then $|F| \leq |E - F|$.
Proof: Let S be a minimum pendant edge dominating set in G. Take, $F = \{e_1, e_2, e_3, \ldots, e_k\}$ and $|F| = k$. Note that, every vertex of Fhas at least one neighbor in $E - F$. Then, k vertices of F are adjacent to at least k vertices of $E - F$. That is, $E - F \geq k = |F|$. Therefore, $|F| \leq |E - F|$.

Theorem 3.2: For any connected graph G of order n then , $\gamma'_{epe}(G) \leq \frac{m}{2}$

Proof : Let, $F = \{e_1, e_2, e_3, \dots, e_k\}$ be a minimum exact pendant edge dominating set. Then, $\gamma'_{epe}(G) = k$. For m number of edges in a graph $m = |E - F| + |F| = |E - F| + k$. Therefore, $|E - F| = m - k$. We know that, if F is a minimum exact pendant edge dominating set in G then $|F| \leq |E - F|$. Therefore, $|F| \leq m - k$. Since, $|F| = k$ implies $k \leq m - k$. So, $k \leq \frac{m}{2}$ yields that $\gamma'_{epe}(G) \leq \frac{m}{2}$.

Theorem 3.3: For any graph G of order n we have, $2 \leq \gamma'_e(G) \leq \gamma'_{epe}(G) \leq m.$

Further, $\gamma'_{epe}(G) = 2$ if and only if G contains an edge of degree at least $n - 2.$

Proposition 3.4. Let G be any graph with n vertices. Then $\gamma'_{epe}(G) = m$ if and only if G is a path $P_{3.}$

Proposition 3.5. Let G be a connected graph of size m.Then $\gamma'_{epe}(G) = m - 1$if and only if G is P_4

Let $\mathbb{G}$ be the collection of graphs of following types. A cycle and path graph each of order 4 and a path of order 5.

corollary 3.6. Let G be a connected graph of order n.Then $\gamma'_{epe}(G) = n - 2$if and only if $G \in \mathbb{G}$.

Theorem 3.7. Let G be any graph, Then $\left\lceil \frac{m}{1+\Delta_e(G)} \right\rceil \leq \gamma'_{epe}(G) \leq m - \Delta'_e(G) + 1.$

Proof. Let G be any graph.

Since any edge in G can dominate at most $\Delta'_e(G) + 1$ edges including itself, it follows that $\left\lceil \frac{m}{1+\Delta'_e(G)} \right\rceil \leq \gamma'_{epe}(G)$. On the other hand, let e be a vertex of maximum degree $\Delta'_e(G)$ in G. Then for any vertex $v \in N_e(u)$, the set $(V - N_e(u)) \cup \{v\}$ will be a exact pendant edge dominating set in G. Therefore $\gamma'_{epe}(G) \leq m - \Delta'_e(G) + 1$

Theorem 3.8: For any graph G of order n, $\gamma'_{epe}(G) + \gamma'_{epe}(\bar{G}) \leq m + 2.$

Proof: Let G be any graph. Suppose G contains an isolated edge then $\gamma'_{epe}(G) \leq m$ and $\gamma'_{epe}(\bar{G}) \leq 2.$ Therefore $\gamma'_{epe}(G) + \gamma'_{epe}(\bar{G}) \leq m + 2.$ Similarly, if $\bar{G}$ contains an isolated edge, we obtain that, $\gamma'_{epe}(G) + \gamma'_{epe}(\bar{G}) \leq m + 2.$

Suppose G and $\bar{G}$contains no isolated edges and also, we know that if a graph G has no isolated edges, then $\gamma'_{epe}(G) \leq \frac{m}{2}$ also $\gamma'_{epe}(\bar{G}) \leq \frac{m}{2}$.

Since, $\gamma'_{epe}(G) \leq \gamma'_{pe}(G) + 1$ always, it follows that $\gamma'_{epe}(G) \leq \frac{m}{2} + 1$and $\gamma'_{epe}(\bar{G}) \leq \frac{m}{2} +$

Therefore, $\gamma'_{epe}(G) + \gamma'_{epe}(\bar{G}) \leq 2\left(\frac{m}{2}\right) + 2 = m + 2$. Therefore, $\gamma'_{epe}(G) + \gamma'_{epe}(\bar{G}) \leq n + 2.$

Theorem 3.9. Let G be a graph with $\Delta' < m - 1. Then\ \gamma'_{epe}(G) \leq m - \Delta'.$

Proof. Let $e \in E(G)\ and \deg e = \Delta'$. Since G is connected and $\Delta' < m - 1$, there exist two adjacent edges$e_1\ and\ e_2$ and w such that$e_1 \in N(e)$ and $e_2 \notin N[e]$. Now, let $X = (N(e) - \{e_1\}) \cup \{e_2\}$. Clearly $E - X$ is a exped-setof G and hence $\gamma'_{epe}(G) \leq m - \Delta'$.

Theorem 3.10. Let G be a graph such that $\mathrm{E} = (e_1, e_2, e_3, \dots, e_n)$, $\deg(e_i) \geq 1$ and G have k pendant edges. Then $\gamma'_{epe}(G) \leq m - k$

Proof. Let Y be the set of all pendant edges of a graph G. Let $|Y| = k$. Then $(E - Y)$ is ncped-set of G. Hence $\gamma'_{epe}(G) \leq m - k.$

Theorem 3.11. For any graph G $\gamma'_{epe}(G) \leq \left\lfloor \frac{3m}{4} \right\rfloor$.

Proof: Let X be a maximum matching of the graph G. Mark the edges of X by $e_1, e_2, e_3, \dots, e_k, e_{k+1}, \dots, e_r$ such that the edges $e_i\ and\ e_{i+1}, i\ is\ oddand\ 1 \leq i \leq k - 1$ are adjacent to common edge f(e_i) with maximum value of k. Let $Y = \{f(e_i): i\ is\ odd\ \}$. Then $X \cup Y$ is an exact pendant edge dominating set with $\langle N(X \cup Y) \rangle$ is connected and $\gamma'_{epe}(G) \leq |X \cup Y| \leq \left\lfloor \frac{3m}{4} \right\rfloor$.

Theorem 3.12. For any non-trivial graph G, $\gamma'_{epe}(G) + \chi'(G) \leq 2m - 1.$

Proof. If $\gamma'_{ncpe}(G) + \chi'(G) = 2m$, then $\gamma'_{ncpe}(G) = m$ and $\chi'(G) = m$, then G is a complete graph with $\gamma'_{ncpe}(G) = m$, which gives G is trivial and hence

$$\gamma'_{epe}(G) + \chi'(G) \leq 2m - 1.$$

Remark.(i) Let G be a graph with $\gamma'_{epe}(G) + \chi'(G) = 2m - 1$. Iff G is isomorphic to k_3.

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