

# Improving the Decoding of SVD-SCMA Signals through Matrix Transformations

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**Abstract**—This work proposes to use the properties of the Moore-Penrose pseudo inverse (PINV), to pre-process the information received in a 5G and Beyond communications system (B5G), and in this way improve the decoding of the signals even in the presence of a channel affected by Rayleigh fading channel.

**Index Terms**— Singular Value Decomposition, Pseudo-inverse matrix, SCMA, 5G, machine learning.

## I. INTRODUCTION

Beyond 5G (B5G) communications are intended to deliver extremely fast data transmission rates, extremely high capacity, extremely low latency, multiple terminal connections, and extremely low power consumption, along with increase in the scope of connectivity over land, sea, and space too, this would have the power to reshape the world from an Internet of Things (IoT) to an Internet of Everything. For fifth generation and beyond communication systems (B5G), utilises non-orthogonal multiple access (NOMA), which allows controllable interference through the allocation of non-orthogonal resources, increasing the complexity of the receiver [1-3]. Sparse code multiple access (SCMA) is deployed in mobile communications networks beyond fifth generation (5G) [4]. The SCMA uses low-density codebooks to encrypt user bits into multidimensional code words, thus User data is transmitted via several radio subcarriers. Despite with several SCMA codebook design methods been created [5-6], designing an optimal codebook is an ongoing topic within the scholarly community.

## II. RELATED WORK AND CONTRIBUTIONS

SCMA encodes two user bits into low-density multidimensional codewords [6]. Recently, to improve the design of SCMA codebooks, suggested in [7] to employ SVD to optimize the distance between points of the IQ constellation. Later in [8] it is proposed to use 8-APSK to enhance the spectral efficiency, transmitting 3 user bits simultaneously. With two user bits, you have an IQ constellation of 64 points, and with three bits you have 512 possible combinations to represent the data. Machine learning has recently been used for decoding both optical [9] and SCMA signals [10], this is way in this work machine learning is used for two- and three-bits decoding.

In the other hand, probably the most often used Moore-Penrose pseudo-inverse (PINV) relates to the least

squares method used in various areas of science. Other, frequently interdisciplinary, study fields that utilize PINV include diffuse optical imaging, robotics, inverse problems digital image restoration, and neural networks [11]. This work proposes a method to preprocess the received information using the PINV. This matrix transformation is applied before the signal enters the machine learning neural network. The results obtained through simulations shows an improvement in signal decoding of the projected communications system. Following a brief description of the properties of the singular value decomposition (SVD), the reception of signals, and the PINV, the simulation scenario and the neural network configuration parameters are described. A discussion of the preliminary findings and the work's conclusions are then provided.

### III. SINGULAR VALUE DECOMPOSITION TO ENCODE SCMA SIGNALS

In [7], it is proposed to use SVD to encode user data. Broadly speaking, a real or complex  $n \times m$  rectangular matrix is factorized into three factors for SVD, where  $n \times m$  is:

$$A_{n \times m} = U \Sigma V^* \quad (1)$$

where:

$U$  is a unitary matrix of  $n \times p$ ,

$V^*$  is a unitary matrix of  $m \times p$ ,

$\Sigma$  is a diagonal matrix of  $p \times p$  with non-negative values sorted in its descending order and

$p$  is the rank of the matrix  $A$ .

So, equation (1) can be rewritten as:

$$A_{n \times m} = U_{n \times p} \Sigma_{p \times p} V_{m \times p}^* \quad (2)$$

The  $\sigma_i$  values of  $\Sigma$  being the singular values of  $A$ , determines the rank of the matrix  $A$ . The right singular vectors are known as the  $V$  columns while the left singular vectors are known as the  $U$  columns. When SVD is used to create SCMA codebooks, the matrices that are created are orthogonal along with orthonormal vectors, resulting in IQ constellations with the ideal amount of space between neighboring points and shielding signals from channel fading.

### IV. SIGNAL RECEPTION AND THE PINV

The architecture of the codebooks and the channel noise have a direct impact on the receiver's ability to recognize and decode SCMA codewords. For detection and decoding, the Message Passing Algorithm (MPA) and minimal Euclidean distance are employed. For uplink detection and decoding of the received signal, the authors of [8, 10] applied supervised learning techniques employing neural networks. As mentioned previously, this work proposes to pre-process the received signal using PINV before the decoding module using machine learning.

From the SVD of a complex  $n \times m$  rectangular matrix  $A$ , it is possible to establish a formula to calculate the PINV ( $A^\dagger$ ).

$$\begin{aligned} \text{If } A_{n \times m} &= U \Sigma V^* \\ \text{Then, } A^\dagger &= V \Sigma^\dagger U^*, \end{aligned} \quad \begin{aligned} (3) \\ (4) \end{aligned}$$

$$\text{where } \Sigma^\dagger = \text{diag}(\sigma_1^{-1}, \dots, \sigma_r^{-1}, 0, \dots, 0) = \begin{pmatrix} \Sigma_r^{-1} & 0 \\ 0 & 0 \end{pmatrix}$$

As can be seen, SVD is closely related to PINV, since PINV can be calculated from the singular values. Based on the above, the following is proposed for the pre-processing of the received signal:

If  $X$  the signal that is transmitted after applying SVD, then,  $Y = X + G$  the received signal (original signal plus noise), where  $G$  is the noise matrix with normal distribution and signal-to-noise ratio between -20dB y +20dB. Pre-processing the received signal, a filter given by  $F = XY^\dagger$  is applied, so the signal that is delivered to the machine learning module is given by:

$$FY = XY^\dagger Y \quad (5)$$

The scenario for the computational simulation is described in the section that follows.

## V. SIGNAL RECEPTION

The SCMA codewords' detection and decoding at the receiver are directly influenced by the codebooks' design as well as the parameters of the communications channel (noise, attenuation, interference). Figure 1 shows the test scenario used.

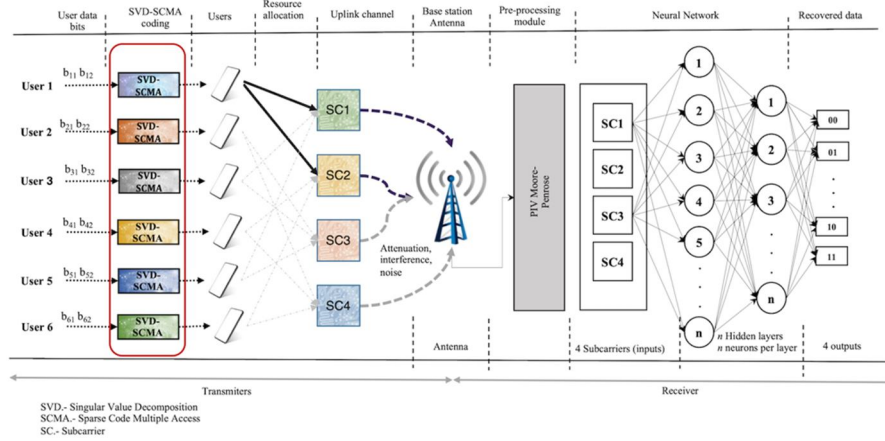


Figure 1. Simulation Scenario

For this test scenario, six users and four radio carriers (SC) are proposed. Users are shown on the left side of the diagram, and their data is divided into pairs that are then encoded using the SVD-SCMA method proposed in [7] and assigned to radio subcarriers. This signal is subsequently transmitted to the uplink of the communications system and is recognized as a Rayleigh fading channel-affected medium. The received signal is pre-processed at the base station using the PIV method proposed in this work, to subsequently be sent to the decoding module that uses machine learning as proposed in [8, 10], the output of the neural network is the data retrieved in pairs of bits.

Since subcarrier 1 transmits data from three users, each bit pair for user 1 can have up to 16 different representations because the value of a bit pair for user 1 must be added to the four probable values for user 2 and the four probable values for user 4, which results in four pairs of bits that can have up to 64 different representations. Subcarriers 1 and 3, which transmit data from users 1, 2, 4 and 1, 3, 5, respectively, are depicted in Figure 5 as constellations in the IQ plane.

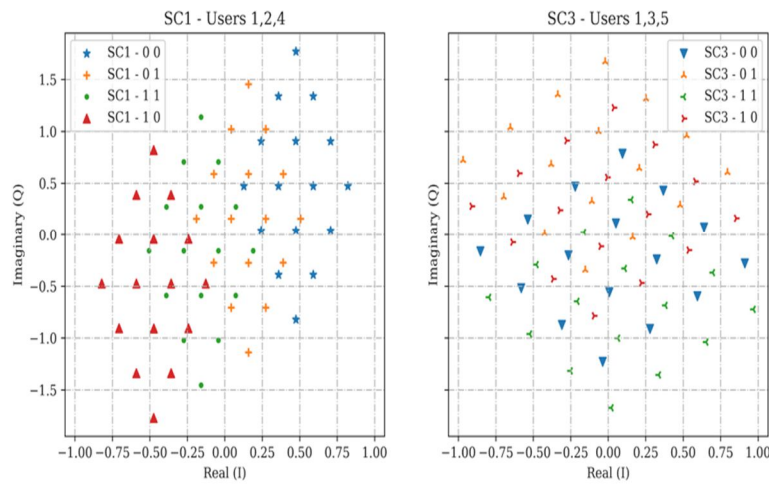


Figure 2. IQ constellation

Table I displays the neural network's configuration parameters.

TABLE I. NEURAL NETWORK CONFIGURATION

| Inputs                                   | 4 (Subcarriers)                                         |
|------------------------------------------|---------------------------------------------------------|
| Outputs                                  | 4 (2 bits)                                              |
| Dense layers                             | 6 (1024, 512, 256, 128, 64 and 32 neurons)              |
| Optimization algorithm                   | Adaptive Moment Estimation (Adam) [24]                  |
| Batch size                               | 50                                                      |
| Validation interval                      | 0.08                                                    |
| $\beta_1$                                | 0.9                                                     |
| $\beta_2$                                | 0.999                                                   |
| learning rate of $l_r$                   | 0.002                                                   |
| Activation function                      | Relu (except for the last layer where SOFTMAX was used) |
| Learning precision of the training stage | 95%                                                     |

The neural network was trained with data that are affected by Rayleigh-type fading with a signal-to-noise ratio between -20dB and +20dB and with 100 iterations to complete the training.

## VI. SIMULATION RESULTS AND DISCUSSION

The new data is generated after the neural network is trained; Figure 3 shows the reference constellation, the received data considering the effects of the Rayleigh fading channel with an SNR = 9 dB.

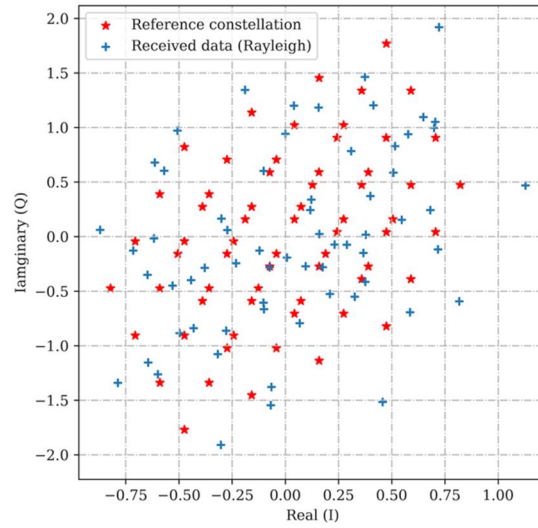


Figure 3. IQ constellation for reference and received data

To facilitate the visualization of the data, Figure 4 shows the first 16 data considering the reference constellation, the received data and the data pre-processed with the Moore-Penrose pseudo-inverse.

From figure 4, it is observed that the data that were pre-processed with the PINV appear closer to the reference constellation than those data that were not processed with the PINV. Figure 5 shows the error bit rate analysis for both the received data and those that were pre-processed with the PINV.

The preliminary results show that there is an improvement in the decoding of the data, when the received signal is pre-processed using PINV, the data were compared against those obtained in previous works.

## VII. CONCLUSIONS

It is recommended to employ PINV to apply it to the signal received from a 5G and beyond communications system (B5G). The results show that the data that is pre-processed appear closer to its reference constellation than those who will not undergo this matrix transformation.

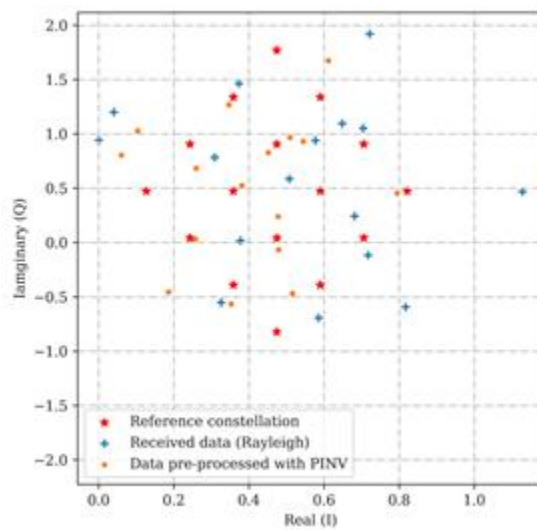


Figure 4. IQ constellation for reference, received data, and pre-processed data

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