

Optimal Control System Design and Stability Analysis of Yaw Control

Pavithra A C¹ and Archana N V²

¹Research Scholar, NIEIT, Mysore and Assistant Professor, Electronics & Communication Dept., ATME College of Engineering, Mysore, Karnataka, India 570 028
Email: pavithraac26@gmail.com

²Professor & Head, Electrical & Electronics Dept., NIE Institute of Technology, Mysore, Karnataka, India 570 028
Email: archana@nie.ac.in

Abstract—Presently, advanced control theory plays a major role in the aircraft system for the control of Yaw, Roll and Pitch angles, which was very much necessary for the stabilization of aircraft system. Following the works of control system community for aircraft applications this paper concentrates on the control of Yaw angle by designing various optimal Linear Quadratic Regulator (LQR) controllers using standard existing tools. The optimal controllers are designed for the aircraft lateral Yaw dynamics and the performance of each optimal LQR feedback controllers are tested and validated using MATLAB/SIMULINK environment and the results are compared with different system conditions to select the best optimal LQR feedback controllers in future aircraft control system.

Index Terms— LQR, Optimal, Yaw Control, Multi Stage, Genetic Algorithm.

I. INTRODUCTION

The modern aircraft system requires advanced automatic control to monitor the aircraft subsystems, particularly for the application of military and civil aviation. The architecture, operations and capacity, etc., of aircraft systems are rapidly changing day to day in the present scenario. Advanced aircraft has various control structures, among them primary and mandatory flight controls are pitch, roll and yaw control which are basically exist in deflection of elevators, ailerons and rudder or combinations of them. The present paper concentrates on the control system design and stability analysis of yaw control (rotation around the vertical axis) with rudder control input by implementing the four different optimization techniques for the lateral dynamics of aircraft system and the results are compared with respect to peak overshoots and settling time.

A. Related Works

A brief literature survey on the control of aircraft system, which will give strong foundation on the existing control techniques for the control and stability analysis is as follows:

The authors in [1] proposed an optimal control algorithm which reduces the error compared to reference value for the yaw angle control in subway systems considering curved road driving. The novel control allocation method was developed in [2] with two optimization objectives, where the results obtained are negligible differences with respect to aerodynamic efficiency. Generalized Dynamic Inversion (GDI) control technique [3] for the linear state dynamic equations of yaw and roll axes control and its effectiveness is verified through

numerical simulations. The authors in [4] scrutinized the aerodynamic change of adding a yaw-wise rotational degree of freedom to a single slotted flap of airplane via computational fluid dynamic analyses and the outcome reveals that a suitable gap has to be matched for ameliorate the lift further. The concurrent approach for the state variables associated in lateral dynamics such as yaw rate and slide slip angle were proposed by the authors in [5] and the simulation results show that the proposed controller yields tire-road friction adaptation with all the considered feedback controllers.

The authors in [6] designed an optimal control model for the reduction in noise and successfully applied for the two aircraft system and the results show the reduction of noise at reception points. In [7], analyzed the control navigation strategy of small Unmanned Aerial Vehicle (UAV) flight control system using time domain time constant specifications ζ & W_n . The simulation results of the proposed method relatively low cost and easy operation which was suitable for static stability. The authors in [8] proposed pole placement technique control based on LQR for the linearized aircraft model. The simulation results of the proposed control techniques show that it will be suitable for small aircraft system.

Addresses the Elevators and Ailerons of two control surfaces in [9] namely Elevators and Ailerons for controlling longitudinal and roll control movement. The control surfaces were modeled and implemented with different intelligent controllers such as optimization techniques Genetic Algorithm & Particle Swarm Optimization. The simulation results performance of the proposed control techniques was evaluated based on time response specification of controllers. In [10] developed the model of an aircraft roll control system which will be useful for control strategy to an actual aircraft system was designed for Matlab/Simulink environment. The state and output equations with time domain methods for the automatic flight control system was proposed by authors in [11] with flight control system considering reference aircraft CHARLIE under different flight conditions.

II. MODELLING OF YAW CONTROL SYSTEM

Presently, aircraft system has two types of dynamical equations; one is lateral and other is longitudinal where both represents the dynamics of aircraft with respect to lateral and longitudinal axis respectively. The state variables yaw, roll and slideslip motions comes under the first category of lateral dynamics [12]. Where, the longitudinal dynamics includes the pitch motions. The current section explains the modeling of Yaw control system. Figure 1, represents the control surfaces of aircraft and the forces, moments and velocity components respectively.

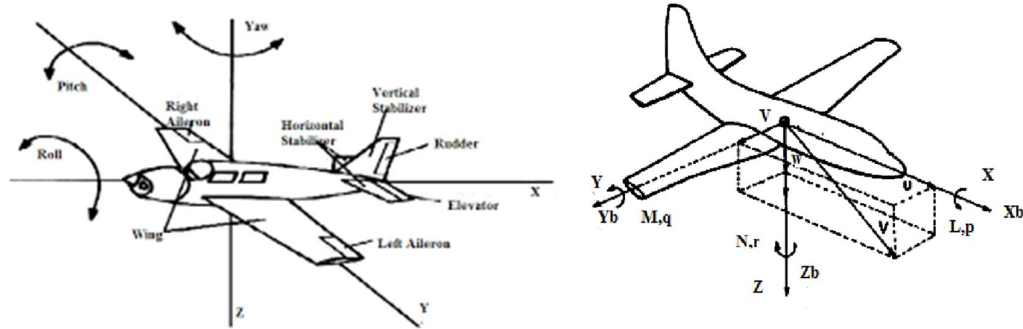


Fig. 1. Aircraft Motions: Yaw, Roll, Pitch and Definitions of force, moments and velocity components in a body fixed frame

The Lateral equations of the aircraft system in state space form is as follows:

$$\dot{x} = Ax + Bu \quad x = \begin{bmatrix} \Delta\beta \\ \Delta P \\ \Delta r \\ \Delta\theta \end{bmatrix} \quad u = \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix}$$

Where, δ_a , δ_r : aileron & rudder deflection; β , θ : sideslip & roll angle; P , r : roll & yaw rate

For, the current research, rudder deflection δ_r is considered as control input. The numerical values [12] for the state space matrices is as follows:

$$A = \begin{bmatrix} -0.254 & 0 & -1 & 0.183 \\ -15.969 & -8.395 & 2.19 & 0 \\ 4.549 & -0.349 & -0.76 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad [B\delta r] = \begin{bmatrix} 0 \\ 23.09 \\ -4.613 \\ 0 \end{bmatrix}$$

III. OPTIMIZATION CONTROL TECHNIQUES

In this work the optimal Linear Quadratic Regulator (LQR) controller is implemented for the aircraft Yaw control. Since, LQR will guarantee stability and also compromise between output performance and control cost. The proposed LQR is tuned for the weighting matrices Q & R by applying four optimization algorithms such as (i) Multistage (ii) Genetic Algorithm (GA) (iii) Particle Swarm Optimization (PSO) and (iv) Artificial Bee Colony (ABC).

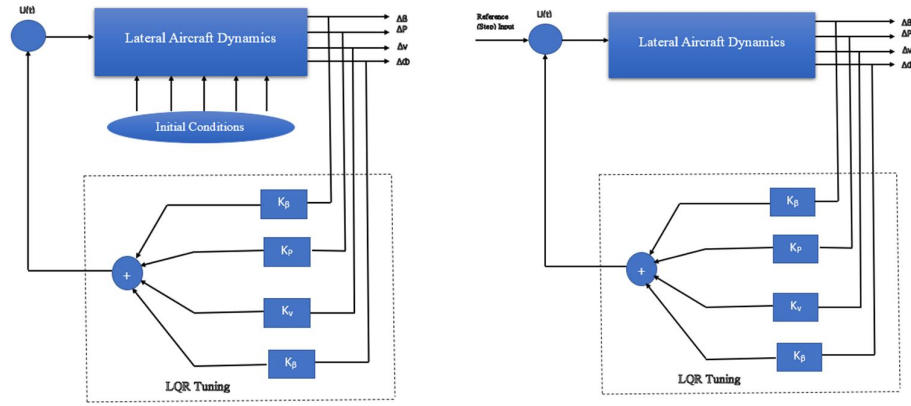


Fig. 2. Aircraft System with Initial Conditions and Reference Inputs

The proposed four optimal LQR tuning algorithms are applied for the aircraft Yaw control system with three flying conditions as shown in Figures 2-3. Figure 2, indicates the flight Yaw control system with initial conditions and the step input as reference input and Figure 3, shows the flight condition with considering both initial conditions as well as reference input. For the sake of completeness, the proposed LQR tuning algorithms are explained in detail:

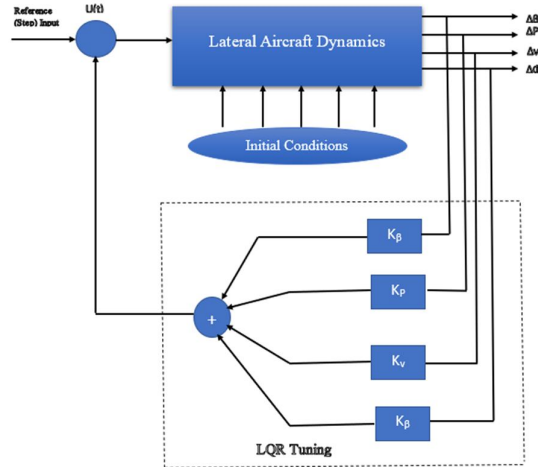


Fig. 3. Aircraft System with Initial Conditions and Reference Input

A. Multistage LQR Algorithm (MS-LQR)

This algorithm was proposed by (R.K. Pandey in 2010) and successfully implemented it for the UPFC based FACTS controllers. The design procedure of this algorithm (currently four stages are considered) is as follows [14]:

1. 1st stage: The weighting matrices Q & R for the initial stage to be considered as Bryson rule

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = [1] \quad [K1, S, E] = \text{lqr}(A, B, Q, R)$$

$$K1 = [0.2096 \quad 0.6711 \quad -0.6348 \quad 1.1047]$$

2. 2nd stage: Choose Q1 & R matrices as

$$Q1 = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = [1] \quad \text{Select, } A1 = A - (B * K1)$$

$$[K2, S, E] = \text{lqr}(A1, B, Q1, R) \quad K2 = [1.6287 \quad 0.3023 \quad -0.6765 \quad 0.4759]$$

3. 3rd stage: Choose Q2 & R matrices as

$$Q2 = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = [1] \quad \text{Select, } A2 = A1 - (B * K2)$$

$$[K3, S, E] = \text{lqr}(A2, B, Q1, R) \quad K3 = [6.4593 \quad 0.1077 \quad -1.4258 \quad 0.3498]$$

4. 4th stage: Choose Q3 & R matrices as

$$Q3 = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = [1] \quad \text{Select, } A3 = A2 - (B * K3)$$

$$[K4, S, E] = \text{lqr}(A3, B, Q1, R) \quad K4 = [22.0275 \quad -0.1000 \quad -2.6207 \quad 0.1862]$$

B. Genetic Algorithm (GA-LQR)

Genetic Algorithm (GA) is one of the optimization techniques which will be useful to find optimal or near-optimal solutions for the optimization problems using the principles of Genetics and Natural Selection. Five phases are considered in a Genetic Algorithm.

- (i) Initial Population;
- (ii) Fitness Function;
- (iii) Selection;
- (iv) Crossover and
- (v) Mutation

The Fitness function for the current paper is defined as follows [15]:

$$F = S.t_r.t_{rmax} + S.ts.t_{smax} + S.O.Mo \quad (1)$$

Where,

F= Fitness Function; t_r = Rise Time; t_{rmax} = Maximum Rise Time; ts = Settling Time; t_{smax} = Maximum Settling Time; O= Overshoot; Mo= Maximum Overshoot.

Table 1, shows the control criterions of the GA optimization algorithm chosen for the current research [15].

TABLE I. CRITERION NUMERICAL VALUES OF THE GA METHOD

GA Criterion	Numerical Value/Approach
Population Size	20
Max No. of generations	100
Selection Method	Normalized Geometric

The weighting matrices Q and R obtained for the GA-LQR controller are:

$$Q = \begin{bmatrix} 2.766661 & 0 & 0 & 0 \\ 0 & 0.010324 & 0 & 0 \\ 0 & 0 & 2.5977 & 0 \\ 0 & 0 & 0 & 157.767 \end{bmatrix} \quad R = [0.00195375]$$

The optimal feedback gain matrix K_{GA} is

$$K_4 = [18.9878 \quad 8.0624 \quad -3.5734 \quad 282.2613]$$

C. Particle Swarm Optimization (PSO-LQR)

This optimization rule was proposed by Eberhart and Kennedy in 1995. The fitness function proposed for the current paper is as follows [15]:

$$F = \omega_{max}(1 - e^{-1(M0+ess)}) + \omega_{min} e^{-(ts-tr)} \quad (2)$$

Where, ω = weighting function and ess = steady state error

The tuning criterion values for the PSO algorithm in MATLAB environment is as in [15]. The weighting matrices Q and R obtained for the PSO-LQR controller are:

$$Q = \begin{bmatrix} 4.31164 & 0 & 0 & 0 \\ 0 & 0.0117533 & 0 & 0 \\ 0 & 0 & 1.62006 & 0 \\ 0 & 0 & 0 & 187.453 \end{bmatrix} \quad R = [0.0011896]$$

The optimal feedback gain matrix K_{PSO} is

$$K_{PSO} = [23.6153 \quad 9.1774 \quad -2.0721 \quad 396.3961]$$

D. Artificial Bee Colony (ABC-LQR):

The Artificial Bee Colony (ABC) algorithm was introduced by Karaboga in 2005 for optimizing numerical problems. The weighting matrices vector Q and R obtained for the ABC-LQR controller are [15]:

$$Q = \begin{bmatrix} 3.64177 & 0 & 0 & 0 \\ 0 & 0.000800173 & 0 & 0 \\ 0 & 0 & 1.63733 & 0 \\ 0 & 0 & 0 & 135.344 \end{bmatrix} \quad R = [0.000149047]$$

IV. SIMULATION RESULTS

The aircraft yaw control system is simulated using the proposed optimal LQR controllers under three cases in MATLAB/SIMULINK environment as shown in Table 3 and the results are compared. The deviation in Yaw rate (Δr) responses for the case (i), case (ii) & case (iii) with the legends MS-LQR, GA-LQR, PSO-LQR & ABC-LQR are shown in Figures 4-5 followed by comparison of settling time and peak overshoots from Tables 3 & 4.

V. DISCUSSION

The depicted Figures 4-5 and Tables 3-4 indicates the comparison of all the four proposed optimal LQR Controllers (MS-LQR, GA-LQR, PSO-LQR & ABC-LQR) for three operating cases of the system. Case (i)

TABLE II. AIRCRAFT YAW CONTROL MATLAB SIMULATIONS

Case	System Operating	LQR Controllers Applied
(i)	Initial Conditions	Multi-stage, GA, PSO & ABC
(ii)	Step Input	Multi-stage, GA, PSO & ABC
(iii)	Initial Conditions and Step Input	Multi-stage, GA, PSO & ABC

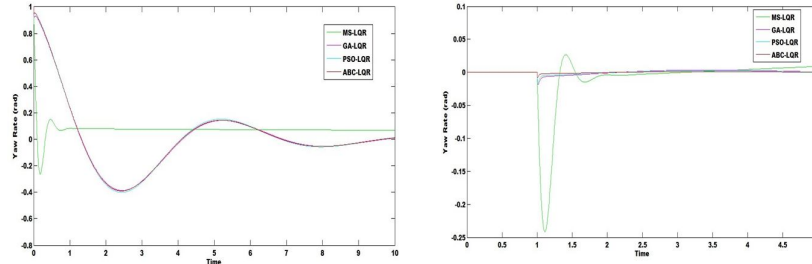


Fig. 4. Yaw Rate Deviation for Case (i) and Case (ii)

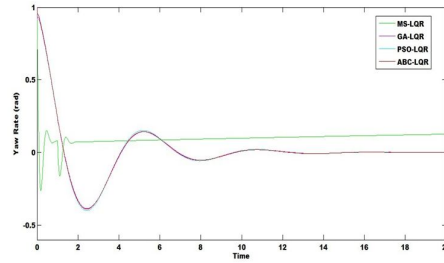


Fig. 5. Yaw Rate Deviation for Case (iii)

TABLE III. COMPARISON OF PEAK OVERSHOOTS (M_p) AND SETTLING TIME (T_s) FOR PROPOSED CONTROL METHODS

Case	(i)	(ii)	(iii)
MS-LQR	-0.25	-0.24	-0.25
GA-LQR	-0.38	-0.018	-0.38
PSO-LQR	-0.4	-0.017	-0.4
ABC-LQR	-0.35	-0.008	-0.35

Case	(i)	(ii)	(iii)
MS-LQR	1s	Above 5s	Above 20s
GA-LQR	Above 10s	3.5s	15s
PSO-LQR	Above 10s	3.55s	15s
ABC-LQR	Above 10s	3s	15s

TABLE IV. COMPARISON OF PEAK OVERSHOOTS (M_p) AND SETTLING TIME (T_s) FOR BEST PROPOSED CONTROL METHOD WITH OTHER CONTROL TECHNIQUES

Case	(i)	(ii)
Proposed MS-LQR	-0.25	-0.25
GDI by [3]	-0.3	-
Bryson-LQR by [12]	-	1.1

Case	(i)	(ii)
Proposed MS-LQR	1s	-
Proposed GA-LQR	-	3.5s
GDI by [3]	Above 5s	-
Bryson-LQR by [12]	-	4s

results, reveals that the Multistage-LQR provides better performance with respect to peak overshoots & settling time compared to other LQR tuning controllers and Case (ii) represents for the system operating with reference input the ABC-LQR controller has good effectiveness both in peak overshoots as well as settling time in damping compared to other optimal LQR controllers. Finally, Case (iii) the system operating with initial conditions as well as reference input represents again the proposed MS-LQR optimal controller provides better peak overshoots but settling time is quite higher compared to other optimal LQR Controllers. Finally, the overall results reveal that for different operating cases of aircraft, if the multi-stage LQR is tuned with further stages by selecting appropriate weighting matrices Q & R results in improved performance compared to other optimization techniques shown in the current research.

VI. CONCLUSION

In the present paper, an aircraft Yaw control system was developed using optimal LQR control technique. The designed controller is validated by simulating using MATLAB/SIMULINK platform under three cases (i) Initial Conditions, (ii) Reference (Step) Input and (iii) Both Initial Conditions and Reference Input to validate and compare the behavior of the proposed control system. In the current paper, the LQR controller was tuned using MS-LQR, GA-LQR, PSO-LQR & ABC-LQR Algorithms. The simulation results concludes that for the case (i) & case (iii) MA-LQR provides better performance with respect to peak overshoots and settling time and for the case (ii) ABC-LQR has more effectiveness in damping compared to other optimization controls.

In the future, further the current research is carried out by implementing Linear Switched Control System to switch between two optimized feedback controllers to utilize the good properties of both sub systems and to obtain a new property which is not present in any of the sub system.

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