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Analysis of an Inventory System with Ramp Type Demand Rate, Partial Shortages under Inflation and Learning

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***Abstract*—The main purpose of this paper is to investigate the optimal replenishment policy under the learning effect and allowable shortages within the economic order quantity (EOQ) framework. We adopt a demand function which is ramp type pattern. The assumption that the goods in inventory always preserve their physical characteristics is unethical. Therefore another important factor is deterioration, as it may yield misleading results. The unit production cost is inversely proportional to the demand rate. Hence a mathematical model has been developed in the view of above scenario, in order to determine the optimal costs for two different cases, by minimizing the present worth of total costs. Finally, numerical examples are provided to illustrate the theoretical results and a sensitive analysis of the optimal solution has been performed to showcase the effect of various parameters.**

***Index Terms*— Ramp Type Demand, Weibull Deterioration, Unit Production Cost, Shortages, inflation, learning.**

I. INTRODUCTION

In recent years, inventory problems for deteriorating items have been widely studied. Deteriorating is a general phenomenon for many products, in which fruits or vegetables are spoiled directly while alcohol physical depletate over time , and electronic products deteriorate rapidly as time went through a gradual of loss of potential utility, that result in the decrease of usefulness of commodities. The first attempt to describe the optimal ordering policies for such items was made by Ghare and Schrader in 1963. Covert and Philip [1973] proposed an inventory model with weibull distribution rate without considering shortages. More related articles can be referred to like Yang et al. [2011], and Singh et al. [2007], Singh C.and Singh S.R. [2011], Manna and Chaudhuri [2006, 2016] and so forth.

However, for deteriorating items it is unethical that the demand rate increases continuously during their full life cycle. Based on such realistic facts, Hill proposed an inventory model with ramp type demand rate. Mandal and Pal [1998] extended this model to allow shortages. Further, Wu [2001] extended it to have Weibull distribution deterioration and time dependent backlogging. Shouri et. al. [2009] also considered a model by introducing a general ramp type demand rate, partial backlogging and Weibull deterioration rate.

While, for some short life cycle products, the demand rate may increase up to a certain level, then reach a stabilized period, and finally decrease when the inventory level falls to zero. There are many other related

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literatures about such inventory model, such as Deng et. al. [2007], Giri et. al. [2003], Singh C.and Singh S.R. [2011].

Furthermore, when shortages occur, some customers are willing to wait for backorders to be fulfilled and others whom are often fickle and increasingly less loyal would not. Therefore, the occurrence of shortages in inventory is a natural phenomenon and in practice shortages are partially backlogged and partially lost. Some related works can be found in Abad [1996], Dye et al. [2007], Wu et al. [2006], Singh et al. [2009], Goyal et al. [2013].

Apart from the above mentioned facts, “learning” as natural phenomena, are observable everywhere. Learning implies that the performance of a system engaged in a repetitive task improves with time. This improvement of the system can be observed in manufacturing companies as a reduction in the cost and/ or time of production. Singh et. al. [2013], Jayshree & Singh [2016], Kumar et al (2013), Yadav et. al. [2012] and many others have developed inventory models to cover this phenomenon.

After the global economic crisis, developing countries have suffered from large scale inflation. However, from a financial point of view, an inventory represents a capital investment and must complete with other assets for a firm’s limited capital funds .Understanding of inflation and time value of money is crucial. To get the real estimate of all costs incurred, it is logical to incorporate the net profit of inflation. The pioneer research in this area was Buzzacott [1975] and Misra [1975]. Thereafter, several interesting research papers have appeared e.g. Yang et. al. [2001], Yang [2012], Singh et. al.[2008, 2009].

This paper incorporates Weibull deterioration and ramp type demand with allowable shortages under learning phenomenon .We extend the work of Jayshree & Jain [2016] to propose an optimal replenishment policy within the EOQ framework and also carry out a sensitivity analysis of the main parameters.

II. Assumptions and Notations

The following notations and assumptions are considered to develop the inventory model

A. Notations

K- Unit Production cost (units /unit time)

X_1 –Holding cost per order is partly constant and partly decreases in each cycle due to learning effect and defined as $X_{01}+\frac{X'_1}{n^k}$, $k > 0$

X_3 – Deterioration cost per order is partly constant and partly decreases in each cycle due to learning effect and defined as $X_{03}+\frac{X'_3}{n^k}$, $k > 0$

X_4 –Shortage cost per order is partly constant and partly decreases in each cycle due to learning effect and defined as $X_{04}+\frac{X'_4}{n^k}$, $k > 0$

X_5 – Lost sales cost per order is partly constant and partly decreases in each cycle due to learning effect and defined as $X_{05}+\frac{X'_5}{n^k}$, $k > 0$

X – Total average cost for a production cycle

r- Inflationary rate

δ – Backlogging rate

B. Assumptions

(1) Demand rate in ramp type function of time, i.e. demand rate R= f (t) is assumed to be a ramp type function of time f(t)= D_0[t-(t-μ) H(t-μ)] , $D_0 > 0$ and H(t) is a Heaviside’s function:
H (t-μ) = $\begin{cases} 1 & if\ t \geq \mu \\ 0 & if\ t < \mu \end{cases}$
(2) Deterioration varies unit time and it is function of two parameter Weibull distribution of the time, i.e. $\alpha\beta t^{\beta-1}$, $0 < \alpha < 1$, $\beta \geq 1$, where t denote time of deterioration .
(3) Lead time is zero.
(4) Inflation is considered.
(5) Learning phenomenon is also considered.
(6) Shortage are Allowed and partially backlogged.
(7) K= γ f (t) is the production rate where γ (> 1) is a constant.

The unit production cost $v = \alpha_1 R^{-s}$ where $\alpha_1 > 0, s > 0$ and s ≠ 2.
α_1 is obviously positive since v and R are both non-negative. Also higher demands result in lower unit cost of production. This implies that v and R are inversely related and hence, must be non-negative i.e. positive.
Now,

$$\frac{dV}{dR} = -\alpha_1 s R^{-(s+1)} < 0.$$

$$\frac{d^2V}{dR^2} = \alpha_1 s(s+1)R^{-(s+2)} > 0.$$

Thus, marginal unit cost of production is an increasing function of R. These results imply that, as the demand rate increases, the unit cost of production decreases at an increasing rate. Due to this reason, the manufacture is encouraged to produce more as the demand for the item increases. The necessity of restriction s≠ 2 arises from the nature of the solution of the problem.

III. Mathematical Formulation of the Model

Case 1 ($\mu \leq t_1 \leq t_2$)
The stock level initially is zero. Production starts just after t=0. When the stock attains a level q at time t= t_1, then the production stops at that time. The time point μ occurs before the point t=t_1, where demand is stabilized after that the inventory level diminishes due to both demand and deterioration ultimately falls to zero at time t = t_2. After time t_2 shortages occurs at t=T, which are partially backlogged and partially lost. Then, the cycle repeats.
Let Q(t) be the inventory level of the system at any time t($0 \leq t \leq t_2$). The differential equations governing the system in the interval $[0,t_2]$are given by

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = K - F(t) \qquad 0 \leq t \leq \mu \qquad (1)$$

with the condition Q(0)=0

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = K - F(t) \qquad \mu \leq t \leq t_1 \qquad (2)$$

with the condition $Q(t_1) = q$

$$\frac{dQ(t)}{dt} + \propto \beta t^{\beta-1} Q(t) = -F(t) \qquad t_1 \leq t \leq t_2 \qquad (3)$$

with the condition $Q(t_1) = q, Q(t_2)=0$

$$\frac{dQ(t)}{dt} = -e^{-\delta(T-t_2)} F(t) \qquad t_2 \leq t \leq T \qquad (4)$$

with the condition $Q(t_2)=0$
Using ramp type function F(t), equation (1),(2),(3),(4) become respectively

$$\frac{dQ(t)}{dt} + \propto \beta t^{\beta-1} Q(t) = (\gamma-1)D_0 t \qquad 0 \leq t \leq \mu \qquad (5)$$

with the condition Q(0) = 0

$$\frac{dQ(t)}{dt} + \propto \beta t^{\beta-1} Q(t) = (\gamma-1)D_0 \mu \qquad \mu \leq t \leq t_1 \qquad (6)$$

with the condition $Q(t_1) = q$

$$\frac{dQ(t)}{dt} + \propto \beta t^{\beta-1} Q(t) = D_0 \mu \qquad t_1 \leq t \leq t_2 \qquad (7)$$

With the conditions $Q(t_1)$= q, $Q(t_2)$= 0,

$$\frac{dQ(t)}{dt} = -e^{-\delta(T-t_2)} D_0 \mu \qquad t_2 \leq t \leq T \qquad (8)$$

with the condition $Q(t_2)=0$
(5),(6),(7),(8) are first order linear differential equations
For the solution of equation (5) we get

$$Q(t)e^{\alpha t^\beta} = (\gamma-1)\int D_0 t e^{\alpha t^\beta} + C$$

$$= (\gamma-1)D_0 \left[\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} + \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)} + --\right] + C \qquad (9)$$

By using the condition Q (0) = 0 (10)

$$Q(t) = (\gamma-1)D_0 e^{-\alpha t^\beta}\left[\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{(\beta+2)} + \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)} + --\right], 0 \leq t \leq \mu \qquad (11)$$

for the solution of equation (6) we have

$$\int_{\mu}^{t} d[e^{\alpha t^{\beta}} Q(t)] = (\gamma-1)\, D_0\, \mu \int_{\mu}^{t} e^{\alpha t^{\beta}} dt$$

$= (\gamma-1)D_0\mu e^{-\alpha t^{\beta}}\ [t-\frac{\mu}{2}+\frac{\alpha t^{\beta+1}}{(\beta+1)}+\frac{\alpha^2 t^{2\beta+1}}{2(2\beta+1)}-\frac{\alpha\mu^{\beta+1}}{(\beta+1)(\beta+2)}-\frac{\alpha^2\mu^{2\beta+1}}{2(2\beta+1)(2\beta+2)}], \mu \le t \le t_1$ (12)

The solution of equation (7) is given by

Q(t) $e^{\alpha t^{\beta}} = -Do\mu\left(t+\frac{\alpha t^{\beta+1}}{\beta+1}+\frac{\alpha^2 t^{2\beta+1}}{2(2\beta+1)}+\quad\right)+C$

Putting $Q(t)_1 = q$ we get

$C=qe^{\alpha t_1^{\beta}} + Do\mu\left(t_1+\frac{\alpha t_1\beta+1}{\beta+1}+\frac{\alpha^2 t_1^{2\beta+1}}{2(2\beta+1)}+\quad\right)$ (13)

Using initial condition $Q(t_2)=0$ in equation (13) we have,

q= $D_0\,\mu e^{-\alpha t_1^{\beta}}\,(t_2+\frac{\alpha t_2{}^{\beta+1}}{\beta+1}+\frac{\alpha^2 t_2{}^{2\beta+1}}{2(2\beta+1)}+\quad)-D_0\mu\, e^{-\alpha t_1^{\beta}}\,(t_1+\frac{\alpha t_1{}^{\beta+1}}{\beta+1}+\frac{\alpha^2 t_1{}^{2\beta+1}}{2(2\beta+1)}+--)$

Substitute q in equation (13) the solution of equation (7) is

$Q(t) = D_0\,\mu\, e^{-\alpha t^{\beta}}[(t_2-t)+\frac{\alpha}{\beta+1}(t_2{}^{\beta+1}-t^{\beta+1})+\frac{\alpha^2}{2(2\beta+1)}(t_2{}^{2\beta+1}-t^{2\beta+1})+--]$

$t_1 \le t \le t_2$ (15)

The solution of equation (8) is

$\frac{dQ(t)}{dt} = -D_0\mu\, e^{-\delta(T-t_2)}$ $\qquad t_2 \le t \le T$

with boundary condition $Q(t_2) = 0$, we get

$Q(t) = D_0\,\mu[(t_2-t)-\delta(T-t_2)(t_2-t)]$ (16)

Shortage cost over the period [0,T] is defined as

$= -D_0\,\mu\left[\left\{Tt_2-\frac{t_2^2}{2}-\frac{T^2}{2}-\delta\left(\frac{3t_2T^2}{2}-\frac{3t_2^2T}{2}-\frac{T^2}{2}+\frac{t_2^3}{2}\right)-\left\{r\frac{T^2t_2}{2}+r\frac{t_2^3}{2}+r\frac{T^3}{3}+\delta r\,(T-t_2)\left(\frac{t_2T^2}{2}-\frac{t_2^3}{6}-\frac{T^3}{3}\right)\right\}\right]$ ` (17)

Lost sales cost per cycle is

LS= $D_0\,\mu\int_{t_2}^{T}(1-e^{-\delta(T-t_2)})\,dt$

LS= $D_0\,\mu\int_{t_2}^{T}\delta(T-t_2)(t_2-t)\,dt$

Lost sale cost over the period [0,T] is given by

LS= $D_0\,\mu\delta\left[\frac{3T^2t_2}{2}-\frac{3t_2^2T}{2}-\frac{T^3}{2}+\frac{t_2^3}{2}-\frac{5rt_2T^3}{6}-\frac{rt_2^4}{6}+\frac{rTt_2^3}{6}+\frac{rT^4}{3}+\frac{rt_2^2T^2}{2}\right]$ (18)

The total inventory over the period [0, t_2] is

$$\int_0^{t_2} Q(t)dte^{-rt} = \int_0^{\mu} Q(t)e^{-rt}dt + \int_{\mu}^{t_1} Q(t)e^{-rt}dt + \int_{t_1}^{0} Q(t)e^{-rt}dt$$

$$\int_0^{\mu} Q(t)e^{-rt}dt = \int_0^{\mu}(\gamma-1)D_0e^{-\alpha t^{\beta}}[\frac{t^2}{2}+\frac{\alpha t^{\beta+2}}{(\beta+2)}+\frac{\alpha^2t^{2\beta+2}}{2(2\beta+2)}+--]e^{-rt}dt$$

$= (\gamma\text{-}1)\, D_0\ [\frac{\mu^3}{6}-\frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)}-\frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(\beta+2)(2\beta+2)(2\beta+3)}-\frac{r\mu^2}{8}+\frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)}-\frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{2(\beta+2)(2\beta+2)(2\beta+4)}]$

$\int_{\mu}^{t_1} Q(t)e^{-rt}dt = D_0\,\mu(\gamma-1)\int_{\mu}^{t_1}[t-\frac{\mu}{2}+\frac{\alpha t^{\beta+1}}{(\beta+1)}+\frac{\alpha^2t^{2\beta+1}}{2(2\beta+1)}-\frac{\alpha\mu^{\beta+1}}{(\beta+1)(\beta+2)}-\frac{\alpha^2\mu^{2\beta+1}}{2(2\beta+1)(2\beta+2)}-\alpha t^{\beta+1}+\frac{\alpha\mu t^{\beta}}{2}-\frac{\alpha^2t^{2\beta+1}}{(\beta+1)}+\frac{\alpha^2t^{\beta}\mu^{\beta+1}}{(\beta+1)(\beta+2)}]\,e^{-rt}\,dt$

$= D_0\mu\,(\gamma-1)\int_{\mu}^{t_1} t-\frac{\mu}{2}+\frac{\alpha t^{\beta+1}}{(\beta+1)}+\frac{\alpha^2t^{2\beta+1}}{2(2\beta+1)}-\frac{\alpha\mu^{\beta+1}}{(\beta+1)(\beta+2)}-\frac{\alpha^2\mu^{2\beta+1}}{2(2\beta+1)(2\beta+2)}-\alpha t^{\beta+1}+\frac{\alpha\mu t^{\beta}}{2}-\frac{\alpha^2t^{2\beta+1}}{(\beta+1)}+\frac{\alpha^2t^{\beta}\mu^{\beta+1}}{(\beta+1)(\beta+2)}-rt^2+rt\frac{\mu}{2}-\frac{\alpha rt^{\beta+2}}{(\beta+1)}-\frac{r\alpha^2t^{2\beta+2}}{2(2\beta+1)}+\frac{\alpha r\mu^{\beta+1}t}{(\beta+1)(\beta+2)}+\alpha t^{\beta+2}r-\frac{r\alpha\mu t^{\beta+1}}{2}+\frac{r\alpha^2t^{2\beta+2}}{(\beta+1)}-\frac{r\alpha^2t^{\beta+1}\mu^{\beta+1}}{(\beta+1)(\beta+2)}$

$$\int_{t_1}^{t_2} Q(t)e^{-rt}dt = D_0\mu\int_{t_1}^{t_2}[(t_2-t)+\frac{\alpha}{(\beta+1)}(t_2{}^{\beta+1}-t^{\beta+1})+\frac{\alpha^2}{2(2\beta+1)}\,(t_2{}^{2\beta+1}-t^{2\beta+1})]e^{-\alpha t^{\beta}}\,e^{-rt}dt$$

$=D_0\,\mu[\frac{t_2^2}{2}-t_2t_1+\frac{t_1^2}{2}+\frac{(\alpha)t_2^{\beta+2}\beta}{(\beta+1)(\beta+2)}-\frac{\alpha t_1t_2^{\beta+1}}{(\beta+1)}-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha^2t_2^{2\beta+1}t_1}{2(2\beta+1)}+\frac{\alpha^2t_2^{2\beta+2}(\beta+3)}{2(2\beta+2)(\beta+1)}+\frac{\alpha^2t_1^{2\beta+2}(5\beta+3)}{2(2\beta+1)(2\beta+2)(\beta+1)}+$
$\frac{\alpha t_2t_1^{\beta+1}}{(\beta+1)}-\frac{\alpha^2t_2^{\beta+1}t_1^{\beta+1}}{(\beta+1)^2}-\frac{rt_2^2}{2}+rt_2t_1-\frac{rt_1^2}{2}-\frac{r\alpha t_2^{\beta+2}}{(\beta+1)}+\frac{r\alpha t_2^{\beta+1}t_1}{(\beta+1)}+\frac{r\alpha t_1^{\beta+3}\beta}{(\beta+3)(\beta+1)}-\frac{\alpha^2t_2^{2\beta+2}r}{2(2\beta+1)}+\frac{r\alpha t_2^{\beta+3}(2\beta+3)}{(\beta+1)(\beta+2)(\beta+3)}-$
$\frac{3\alpha^2\beta t_2^{2\beta+3}r}{2(2\beta+1)\,(\beta+2)(2\beta+3)}-\frac{\alpha^2(5\beta+3)\,t_1^{2\beta+3}r}{2(2\beta+1)\,(\beta+1)(2\beta+3)}-\frac{\alpha t_2t_1^{\beta+2}}{(\beta+2)}+\frac{\alpha^2t_2^{\beta+1}t_1^{\beta+2}}{(\beta+1)(\beta+2)}]$

Therefore, the total inventory in [0, t_2] is given by

$\int_0^{t_2}Q(t)e^{-rt}dt=(\gamma-1)\,D_0[\frac{\mu^3}{6}-\frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)}-\frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(\beta+2)(2\beta+2)(2\beta+3)}-\frac{r\mu^2}{8}+\frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)}-$
$\frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{2(\beta+2)(2\beta+2)(2\beta+4)}]+D_0\,\mu\,(\gamma-1)\left[\frac{{t_1}^2}{2}-\frac{\mu t_1}{2}-\frac{\alpha\beta {t_1}^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha\beta\mu^{\beta+2}}{2(\beta+1)(\beta+2)}-\frac{\alpha^2(3\beta+1){t_1}^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+1)}+\right.$
$\frac{\alpha^2\beta\mu^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+2)}-\frac{\alpha\mu^{\beta+1}t_1}{(\beta+1)(\beta+2)}-\frac{\alpha^2\mu^{2\beta+1}t_1}{2(2\beta+1)(2\beta+1)}+\frac{\alpha^2{t_1}^{\beta+1}\mu^{\beta+1}}{(\beta+1)^2(\beta+2)}-\frac{r{t_1}^3}{3}+\frac{r\mu^3}{12}+\frac{r{t_1}^2\mu}{4}+\frac{\alpha r{t_1}^{\beta+3}\beta}{(\beta+1)(\beta+3)}+$
$\frac{r\alpha\beta\mu^{\beta+3}}{2(\beta+1)(\beta+3)(\beta+2)}-\frac{r\alpha^2{t_1}^{2\beta+3}}{2(2\beta+1)(2\beta+3)}+\frac{r\alpha\mu^{\beta+1}{t_1}^2}{2(\beta+1)(\beta+2)}+\frac{r\alpha^2\mu^{2\beta+1}{t_1}^2}{4(2\beta+1)(2\beta+2)}+\frac{r\alpha {t_1}^{(\beta+3)}}{(\beta+3)}-\frac{\alpha\mu r{t_1}^{\beta+2}}{2(\beta+2)}+\frac{\alpha^2r{t_1}^{2\beta+3}}{(\beta+1)(2\beta+3)}-$
$\left.\frac{r\alpha^2{t_1}^{\beta+2}\mu^{\beta+1}}{(\beta+1)(\beta+2)^2}+\frac{r\alpha^2\mu^{2\beta+3}(-7\beta^2-12\beta-4)}{8(\beta+1)(\beta+2)^2(2\beta+3)}+\frac{\alpha\mu t_1^{\beta+1}}{2(\beta+1)}\right]+\;D_0\,\mu[\frac{t_2^2}{2}-t_2t_1+\frac{t_1^2}{2}+\frac{(\alpha)t_2^{\beta+2}\beta}{(\beta+1)(\beta+2)}-\frac{\alpha t_1t_2^{\beta+1}}{(\beta+1)}-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-$
$\frac{\alpha^2t_2^{2\beta+1}t_1}{2(2\beta+1)}+\frac{\alpha^2t_2^{2\beta+2}(\beta+3)}{2(2\beta+2)(\beta+1)}+\frac{\alpha^2t_1^{2\beta+2}(5\beta+3)}{2(2\beta+1)(2\beta+2)(\beta+1)}+\frac{\alpha t_2t_1^{\beta+1}}{(\beta+1)}-\frac{\alpha^2t_2^{\beta+1}t_1^{\beta+1}}{(\beta+1)^2}-\frac{rt_2^2}{2}+rt_2t_1-\frac{rt_1^2}{2}-\frac{r\alpha t_2^{\beta+2}}{(\beta+1)}+\frac{r\alpha t_2^{\beta+1}t_1}{(\beta+1)}$
$+\frac{r\alpha t_1^{\beta+3}\beta}{(\beta+3)(\beta+1)}-\frac{\alpha^2t_2^{2\beta+2}r}{2(2\beta+1)}+\frac{r\alpha t_2^{\beta+3}(2\beta+3)}{(\beta+1)(\beta+2)(\beta+3)}-\frac{3\alpha^2\beta t_2^{2\beta+3}r}{2(2\beta+1)\,(\beta+2)(2\beta+3)}-\frac{\alpha^2(5\beta+3)\,t_1^{2\beta+3}r}{2(2\beta+1)\,(\beta+1)(2\beta+3)}-\frac{\alpha t_2t_1^{\beta+2}}{(\beta+2)}+\frac{\alpha^2t_2^{\beta+1}t_1^{\beta+2}}{(\beta+1)(\beta+2)}]$
(19)

Total number of deteriorated items over the period [0,t_2] is given by
Production in [0,μ]+ Production in [μ,t_1] – Demand in [0,μ] –Demand in [μ, t_2]
$=\gamma\int_o^{\mu}Dote^{-rt}dt+\;\gamma\int_o^{t_1}D_0\,\mu e^{-rt}dt-D_0\int_o^{\mu}te^{-rt}dt-\int_{\mu}^{t_2}Do\mu e^{-rt}dt$

$$=\frac{1}{2}\gamma D_0\mu\left[2t_1-\mu-rt_1^2+\frac{r\mu^2}{3}\right]-\frac{1}{2}D_0\,\mu\left[2t_2-\mu-rt_2^2+\frac{r\mu^2}{3}\right] \tag{20}$$

The cost of production in $[u,u+du]$ *is*

$$\text{Kv}\,du=\frac{\alpha_1\gamma}{R^{s-1}} \tag{21}$$

Hence the production cost over the period [o, t_1] is given by
$\int_o^{t_1}\text{Kv}e^{-ru}du=\int_0^{\mu}\text{Kv}e^{-ru}du+\int_{\mu}^{t_1}\text{Kv}e^{-ru}du$
$=\int_o^{\mu}\frac{\alpha_1\gamma}{R^{s-1}}e^{-ru}du+\int_{\mu}^{t_1}\frac{\alpha_1\gamma}{R^{s-1}}e^{-ru}du$

$$=\alpha_1\gamma {D_0}^{1-s}\left[\frac{(s-1)\mu^{2-s}+(2-s)\mu^{1-s}t_1}{(2-s)}\right]+\alpha_1\gamma r{D_0}^{1-s}[\frac{\mu^{3-s}}{2}-\frac{{t_1}^2}{2}\mu^{1-s}-\frac{\mu^{1-s}}{(1-s)}] \tag{22}$$

The total average inventory cost X is given by

X= Inventory Cost + Deterioration Cost+ Production Cost+ Shortage Cost + Lost Sales Cost

$\text{X}=\frac{1}{t_2}[\,X_1\,\{(\gamma\text{-}1)\,D_0\;(\frac{\mu^3}{6}-\frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)}-\frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(\beta+2)(2\beta+2)(2\beta+3)}-\frac{r\mu^2}{8}+\frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)}-\frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{2(\beta+2)(2\beta+2)(2\beta+4)})+D_0$
$\mu(\gamma\text{-}1)\;(\frac{{t_1}^2}{2}-\frac{\mu t_1}{2}-\frac{\alpha\beta{t_1}^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha\beta\mu^{\beta+2}}{2(\beta+1)(\beta+2)}-\frac{\alpha^2(3\beta+1){t_1}^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+1)}+\frac{\alpha^2\beta\mu^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+2)}-\frac{\alpha\mu^{\beta+1}t_1}{(\beta+1)(\beta+2)}-$
$\frac{\alpha^2\mu^{2\beta+1}t_1}{2(2\beta+1)(2\beta+2)}+\frac{\alpha^2{t_1}^{\beta+1}\mu^{\beta+1}}{(\beta+1)^2(\beta+2)}-\frac{r{t_1}^3}{3}+\frac{r\mu^3}{12}+\frac{r{t_1}^2\mu}{4}+\frac{\alpha r{t_1}^{\beta+3}\beta}{(\beta+1)(\beta+3)}+\frac{r\alpha\beta\mu^{\beta+3}}{2(\beta+1)(\beta+3)(\beta+2)}-\frac{r\alpha^2{t_1}^{2\beta+3}}{2(2\beta+1)(2\beta+3)}+$
$\frac{r\alpha\mu^{\beta+1}{t_1}^2}{2(\beta+1)(\beta+2)}+\frac{r\alpha^2\mu^{2\beta+1}{t_1}^2}{4(2\beta+1)(2\beta+2)}+\frac{r\alpha{t_1}^{(\beta+3)}}{(\beta+3)}-\frac{\alpha\mu r{t_1}^{\beta+2}}{2(\beta+2)}+\frac{\alpha^2r{t_1}^{2\beta+3}}{(\beta+1)(2\beta+3)}-\frac{r\alpha^2{t_1}^{\beta+2}\mu^{\beta+1}}{(\beta+1)(\beta+2)^2}+\frac{r\alpha^2\mu^{2\beta+3}(-7\beta^2-12\beta-4)}{8(\beta+1)\,(\beta+2)^2(2\beta+3)}+$
$\frac{\alpha\mu t_1^{\beta+1}}{2(\beta+1)})+D_0\,\mu(\frac{t_2^2}{2}-t_2t_1+\frac{t_1^2}{2}+\frac{(\alpha)t_2^{\beta+2}\beta}{(\beta+1)(\beta+2)}-\frac{\alpha t_1t_2^{\beta+1}}{(\beta+1)}-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha^2t_2^{2\beta+1}t_1}{2(2\beta+1)}+\frac{\alpha^2t_2^{2\beta+2}(\beta+3)}{2(2\beta+2)(\beta+1)}+$

$$\frac{\alpha^2 t_1^{2\beta+2}(5\beta+3)}{2(2\beta+1)(2\beta+2)(\beta+1)} + \frac{\alpha t_2 t_1^{\beta+1}}{(\beta+1)} - \frac{\alpha^2 t_2^{\beta+1} t_1^{\beta+1}}{(\beta+1)^2} - \frac{rt_2^2}{2} + rt_2 t_1 - \frac{rt_1^2}{2} - \frac{r\alpha t_2^{\beta+2}}{(\beta+1)} + \frac{r\alpha t_2^{\beta+1} t_1}{(\beta+1)} + \frac{r\alpha t_1^{\beta+3}\beta}{(\beta+3)(\beta+1)} - \frac{\alpha^2 t_2^{2\beta+2} r}{2(2\beta+1)} - \frac{r\alpha t_2^{\beta+3}(2\beta+3)}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3\alpha^2 \beta t_2^{2\beta+3} r}{2(2\beta+1)(\beta+2)(2\beta+3)} - \frac{\alpha^2(5\beta+3)\, t_1^{2\beta+3} r}{2(2\beta+1)(\beta+1)(2\beta+3)} - \frac{\alpha t_2 t_1^{\beta+2}}{(\beta+2)} + \frac{\alpha^2 t_2^{\beta+1} t_1^{\beta+2}}{(\beta+1)(\beta+2)})\} + X_3\{\frac{1}{2}\gamma D_0 \mu\left(2t_1 - \mu - rt_1^2 + \frac{r\mu^2}{3}\right) - \frac{1}{2} D_0\, \mu(2t_2 - \mu - rt_2^2 + \frac{r\mu^2}{3})\} - X_4 D_0\, \mu\left\{Tt_2 - \frac{t_2^2}{2} - \frac{T^2}{2} - \delta\left(\frac{3t_2T^2}{2} - \frac{3t_2^2T}{2} - \frac{T^3}{2} + \frac{t_2^3}{2}\right) - r\frac{T^2 t_2}{2} + r\frac{t_2^3}{2} + r\frac{T^3}{3} + \delta r\,(T - t_2)\left(\frac{t_2T^2}{2} - \frac{t_2^3}{6} - \frac{T^3}{3}\right)\right\} + X_5 D_0\, \mu\delta\left(\frac{3T^2t_2}{2} - \frac{3t_2^2T}{2} - \frac{T^3}{2} + \frac{t_2^3}{2} - \frac{5rt_2T^3}{6} - \frac{rt_2^4}{6} + \frac{rTt_2^3}{6} + \frac{rT^4}{3} + \frac{rt_2^2T^2}{2}\right) + \alpha_1\gamma {D_0}^{1-s}\{\frac{(s-1)\mu^{2-s} + (2-s)\mu^{1-s}t_1}{(2-s)}\} + \alpha_1\gamma r {D_0}^{1-s}\{\frac{\mu^{3-s}}{2} - \frac{{t_1}^2}{2}\mu^{1-s} - \frac{\mu^{1-s}}{(1-s)}\}] \quad (23)$$

$$X_1 = X_{01} + \frac{X_3'}{n^k}$$

Where X_1 is continuously deceases over n since $\frac{dX_1}{dn} < 0, n > 0$

$$X_3 = X_{03} + \frac{X_3'}{n^k}$$

Where X_3 is continuously deceases over n since $\frac{dX_3}{dn} < 0, n > 0$

$$X_4 = X_{04} + \frac{X'_4}{n^k}$$

Where X_4 is continuously deceases over n since $\frac{dX_4}{dn} < 0, n > 0$

$$X_5 = X_{05} + \frac{X_5'}{n^k}$$

Where X_5 is continuously deceases over n since $\frac{dX_4}{dn} < 0, n > 0$

Optimum values of t_1 and t_2 for minimum average cost X are the solutions of the equations

$$\frac{\partial X}{\partial t_1} = 0 \; and \; \frac{\partial X}{\partial t_2} = 0$$

Provided they satisfy the sufficient conditions

$$\frac{\partial^2 X}{\partial t_1^2} > 0, \frac{\partial^2 X}{\partial t_2^2} > 0 \text{ and } \frac{\partial^2 X}{\partial t_1^2}\frac{\partial^2 X}{\partial t_2^2} - \left(\frac{\partial^2 X}{\partial t_1 \partial t_2}\right)^2 > 0$$

$\frac{\partial X}{\partial t_1} = 0 \; and \; \frac{\partial X}{\partial t_2} = 0$ gives

$$C_1\left\{D_0\mu(\gamma - 1)\left(t_1 - \frac{\alpha\beta t_1^{\beta+1}}{(\beta+1)} - \frac{\alpha^2(3\beta+1)t_1^{2\beta+1}}{2(2\beta+1)(\beta+1)} - \frac{\alpha\mu^{\beta+1}}{(\beta+1)(\beta+2)} - \frac{\alpha^2\mu^{2\beta+1}}{2(2\beta+1)(2\beta+2)} - \frac{\mu}{2} + \frac{\alpha^2 t_1^{\beta}\mu^{\beta+1}}{(\beta+1)(\beta+2)} - rt_1^2 + \frac{rt_1\mu}{2} + \frac{\alpha r t_1^{\beta+2}\beta}{(\beta+1)} - \frac{r\alpha^2 t_1^{2\beta+2}}{2(2\beta+1)} + \frac{r\alpha\mu^{\beta+1}t_1}{(\beta+1)(\beta+2)} + \frac{r\alpha^2\mu^{2\beta+1}t_1}{2(2\beta+1)(2\beta+2)} + r\alpha t_1^{\beta+2} - \frac{\alpha\mu r t_1^{\beta+1}}{2} + \frac{\alpha^2 r t_1^{2\beta+2}}{(\beta+1)} - \frac{r\alpha^2 t_1^{\beta+1}\mu^{\beta+1}}{(\beta+1)(\beta+2)} + \frac{\alpha\mu t_1^{\beta}}{2}\right) + D_0\mu\left(-t_2 + t_1 - \frac{\alpha t_2^{\beta+1}}{(\beta+1)} - \frac{\alpha\beta t_1^{\beta+1}}{(\beta+1)} - \frac{\alpha^2 t_2^{2\beta+1}}{2(2\beta+1)} + \frac{\alpha^2 t_1^{2\beta+1}(5\beta+3)}{2(2\beta+1)(\beta+1)} + \alpha t_2 t_1^{\beta} - \frac{\alpha^2 t_2^{\beta+1} t_1^{\beta}}{(\beta+1)} + rt_2 - rt_1 + \frac{r\alpha t_2^{\beta+1}}{(\beta+1)} + \frac{r\alpha t_1^{\beta+2}\beta}{(\beta+1)} - \frac{\alpha^2(5\beta+3)t_1^{2\beta+2} r}{2(2\beta+1)(\beta+1)} - \alpha t_2 t_1^{\beta+1} + \frac{\alpha^2 t_2^{\beta+1} t_1^{\beta+1}}{(\beta+1)}\right)\right\} + c_3\gamma\, D_0\mu(1 - rt_1) + \alpha_1\gamma D_0^{1-s}\mu^{1-s} - \alpha_1\gamma D_0^{1-s} t_1 \mu^{1-s} = 0 \quad (24)$$

and

$$X_1 D_o\mu\left(t_2 - t_1 + \frac{\alpha\beta t_2^{\beta+1}}{(\beta+1)} - \alpha t_1\, t_2^{\beta} - \frac{\alpha^2 t_1 t_2^{2\beta}}{2} + \frac{\alpha^2(\beta+3)t_2^{2\beta+1}}{2(\beta+1)} + \frac{\alpha t_1^{\beta+1}}{(\beta+1)} - \frac{\alpha^2 t_1^{\beta+1} t_2^{\beta}}{(\beta+1)} - rt_2 + rt_1 - \frac{r\alpha(\beta+2)t_2^{\beta+1}}{(\beta+1)} + r\alpha t_1 t_2^{\beta} - \frac{r\alpha^2(2\beta+2)t_2^{2\beta+1}}{2(2\beta+1)} + \frac{r\alpha(2\beta+3)t_2^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{3\alpha^2\beta r t_2^{2\beta+2}}{2(2\beta+1)(\beta+2)} - \frac{\alpha t_1^{\beta+2}}{(\beta+2)} + \frac{\alpha^2 t_2^{\beta} t_1^{\beta+2}}{(\beta+2)}\right) - X_3 D_o\mu(1 - rt_2) - X_4 D_o\mu\{(T - t_2) - \delta(\frac{3}{2}T^2 - 3\,t_2 T + \frac{3t_2^2}{2}) - \frac{rT^2}{2} + \frac{3rt_2^2}{2} - \delta r t_2 T^2 + \frac{2}{3}\delta r t_2^3 + \frac{5}{6}\delta r T^3 - \frac{\delta r T t_2^2}{2}\} + X_5 D_o\mu\delta\left(\frac{3}{2}T^2 - 3t_2T + \frac{3}{2}t_2^2 - \frac{5rT^3}{6} - \frac{2}{3}rt_2^3 + \frac{1}{2}rTt_2^2 + rt_2T^2\right) - X = 0 \quad (25)$$

Case–II ($t_1 \leq \mu \leq t_2$)

The production starts with zero stock level at t=0. Production begins at t=0 and continues up to $t=t_1$ and stops as soon as the stock level becomes L at $t= t_2$. Because of reasons of market demand and deterioration of items, the inventory level decreases till it becomes again zero at $t= t_2$. After time $t= t_2$, another important factor occurs which is shortages. After that period, the cycle repeats itself.

Let Q(t) be the inventory level of the system at any time t ($0 \le t \le t_2$). The differential equations governing the system in the interval [0, t_2] are given by

$$\frac{d\,Q(t)}{dt} + \alpha\beta\ t^{\beta-1}\ Q(t) = K - F(t) \qquad 0 \le t \le t_1 \qquad (26)$$

with the condition Q (0) = 0, Q (t_1) = L

$$\frac{d\,Q(t)}{dt} + \alpha\beta\ t^{\beta-1}\ Q(t) = -F(t) \qquad t_1 \le t \le \mu \qquad (27)$$

with the condition Q (t_1) = L

$$\frac{d\,Q(t)}{dt} + \alpha\beta\ t^{\beta-1}\ Q(t) = -F(t) \qquad \mu \le t \le t_2 \qquad (28)$$

with the condition Q (t_2) = 0

$$\frac{d\,Q}{dt} = -e^{-\delta(T-t)}\,\mathrm{F(t)} \qquad t_2 \le t \le T \qquad (29)$$

with the condition Q (t_2) = 0

using ramp type function F (t) equations (26),(27)(28),(29) become respectively

$$\frac{d\,Q(t)}{dt} + \alpha\beta\ t^{\beta-1}\ Q(t) = (\gamma-1)\,D_0 t \qquad 0 \le t \le t_1 \qquad (30)$$

with the condition Q (0) = 0, Q (t_1) = L

$$\frac{d\,Q(t)}{dt} + \alpha\beta\ t^{\beta-1}\ Q(t) = -D_0 t \qquad t_1 \le t \le \mu \qquad (31)$$

with the condition Q (t_1) = L

$$\frac{d\,Q(t)}{dt} + \alpha\beta\ t^{\beta-1}\ Q(t) = -D_0\,\mu \qquad \mu \le t \le t_2 \qquad (32)$$

with the condition Q (t_2) = 0

$$\frac{d\,Q}{dt} = -e^{-\delta(T-t)}\ D_0\mu \qquad t_2 \le t \le T \qquad (33)$$

with the condition Q (t_2) = 0

The solution of equation (30) is given by the expression (11) and we have

$$e^{\alpha t^{\beta}}\ Q(t) = (\gamma-1)\ D_0\left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} + \alpha^2\ \frac{t^{2\beta+2}}{2(2\beta+2)} + \right) + C$$

With the condition Q(0) = 0, we get

$$Q(t) = (\gamma-1)\ D_0\, e^{-\alpha t^{\beta}}\left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} + \frac{\alpha^2\, t^{2\beta+2}}{2(2\beta+2)} + \right) \qquad 0 \le t \le t_1 \qquad (34)$$

Using boundary condition Q(t_1) = L in (34) we get

$$L = (\gamma-1)\ D_0\, e^{-\alpha t_1{}^{\beta}}\left(\frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{\beta+2} + \frac{\alpha^2\, t_1^{2\beta+2}}{2(2\beta+2)} + \right) \qquad (35)$$

Therefore the solution of equation (31) is given by

$$\frac{d\,Q(t)}{dt} + \alpha\beta\ t^{\beta-1}\ Q(t) = -D_0 t$$

$$= -D_0\left(\frac{t^2}{2} + \frac{\alpha\ t^{\beta+2}}{\beta+2} + \frac{\alpha^2\, t^{2\beta+2}}{2(2\beta+2)} + \right) + C$$

Using condition Q(t_1) = L

$$C = Le^{\alpha t_1{}^{\beta}} + D_0\left(\frac{t_1^2}{2} + \frac{\alpha\, t_1^{\beta+2}}{\beta+2} + \frac{\alpha^2\, t_1^{2\beta+2}}{2(2\beta+2)} + \right)$$

$$Q(t) = -D_0 e^{-\alpha t^\beta}\left(\frac{t^2}{2} + \frac{2t^{\beta+2}}{(\beta+2)} + \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)} + \right) + \gamma D_0 e^{-\alpha t^\beta}\left(\frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{(\beta+2)} + \frac{\alpha^2 t_1^{2\beta+2}}{2(2\beta+2)} + \right), \ t_1 \le t \le \mu \qquad (36)$$

Using boundary condition $Q(t_2) = 0$, the solution of equation (32) is given by

$$Q(t) = D_0 \mu e^{-\alpha t^\beta}\left((t_2 - t) + \frac{\alpha}{(\beta+1)}(t_2^{\beta+1} - t^{\beta+1}) + \frac{\alpha^2}{2(2\beta+1)}(t_2^{2\beta+1} - t^{2\beta+2}) + \right), \ \mu \le t \le t_2 \qquad (37)$$

The solution of equation (33) is given by

$$\frac{dQ(t)}{dt} = -D_0\, \mu\, e^{-\delta(T-t_2)} \qquad t_2 \le t \le T$$

By using the boundary condition $Q(t_2) = 0$, we get

$$Q(t) = D_0\, \mu[(t_2 - t) - \delta(T - t_2)(t_2 - t)] \qquad (38)$$

Total inventory over the period $[0, t_2]$ is

$$\int_0^{t_2} Q(t)\, e^{-rt}\, dt = \int_0^{t_1} Q(t)\, e^{-rt}\, dt + \int_{t_1}^{\mu} Q(t)\, e^{-rt}\, dt + \int_{\mu}^{t_2} Q(t)\, e^{-rt}\, dt$$

$$\int_0^{t_1} Q(t)\, e^{-rt}\, dt = (\gamma - 1)\, D_0 \int_0^{t_1} e^{-\alpha t^\beta}\left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} + \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)} + \right) e^{-rt}\, dt$$

$$= (\gamma-1) D_0\left[\frac{t_1^3}{6} - \frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(\beta+2)(2\beta+3)} - \frac{rt_1^4}{8} + \frac{\alpha\beta r t_1^{\beta+4}}{2(\beta+2)(\beta+4)} + \frac{r\alpha^2(3\beta+2)t_1^{2\beta+4}}{2(2\beta+2)(\beta+2)(2\beta+4)}\right]$$

$$\int_{t_1}^{\mu} Q(t)\, e^{-rt}\, dt = D_0 \int_{t_1}^{\mu} e^{-\alpha t^\beta}\left(\frac{\gamma t_1^2}{2} + \frac{\alpha\gamma\, t_1^{\beta+2}}{(\beta+2)} + \frac{\gamma\alpha^2 t_1^{2\beta+2}}{2(2\beta+2)} - \frac{t^2}{2} - \frac{\alpha t^{\beta+2}}{(\beta+2)} - \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)}\right)(1 - rt)$$

$$= D_0 \left[\frac{\gamma\mu t_1^2}{2} - \frac{\gamma t_1^3}{2} + \frac{\alpha\mu\gamma t_1^{\beta+2}}{(\beta+2)} - \frac{\alpha\gamma\beta t_1^{\beta+3}}{2(\beta ss+2)(\beta+1)} + \frac{\alpha^2\gamma t_1^{2\beta+2}\mu}{2(2\beta+2)} - \frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\mu^3}{6} + \frac{t_1^3}{6} + \frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)} - \frac{\alpha\gamma\mu^{\beta+1}t_1^2}{2(\beta+1)} - \frac{\alpha^2\gamma\mu^{\beta+1}t_1^{\beta+2}}{(\beta+2)(\beta+2)} - \frac{\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)} - \frac{\alpha^2\gamma(\beta-2)t_1^{2\beta+3}}{2(\beta+1)(\beta+2)} - \frac{r\gamma\mu^2 t_1^2}{4} + \frac{r\gamma t_1^4}{4} - \frac{r\alpha\gamma t_1^{\beta+2}\mu^2}{2(\beta+2)} - \frac{r\gamma\alpha^2 t_1^{2\beta+2}\mu^2}{8(\beta+1)} + \frac{r\gamma\alpha^2(\beta^2-4\beta-4)t_1^{2\beta+4}}{8(\beta+1)(\beta+2)^2} + \frac{r\mu^4}{8} - \frac{rt_1^4}{8} - \frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)} + \frac{r\alpha\beta t_1^{\beta+4}}{2(\beta+2)(\beta+4)} - \frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{8(\beta+1)(\beta+2)^2} + \frac{r\alpha^2(3\beta+2)t_1^{2\beta+4}}{8(\beta+2)^2(\beta+1)} + \frac{r\alpha\gamma\mu^{\beta+2}t_1^2}{2(\beta+2)} + \frac{r\alpha^2\gamma t_1^{\beta+2}\ \mu^{\beta+2}}{(\beta+2)^2}\right]$$

$$\int_{\mu}^{t_2} Q(t)\, e^{-rt}\, dt = D_0\mu \int_{\mu}^{t_2} e^{-\alpha t^\beta}\left[(t_2 - t_1)\frac{\alpha}{(\beta+1)}\left(t_2^{\beta+1} - t^{\beta+1}\right) + \frac{\alpha^2}{2(2\beta+1)}\left(t_2^{2\beta+1} - t^{2\beta+1}\right)\right] e^{-rt}\, dt$$

$$= D_0\mu\left[\left(\frac{t_2^2}{2} - t_2\,\mu + \frac{\mu^2}{2}\right) - \frac{\alpha\mu t_2^{\beta+1}}{(\beta+1)} + \frac{\alpha\beta t_2^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta\mu^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha^2(\beta-1)t_2^{2\beta+2}}{2(2\beta+2)(\beta+1)} - \frac{\alpha^2\mu t_2^{2\beta+1}}{2(2\beta+1)} - \frac{\alpha^2(3\beta+1)\mu^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+1)} + \frac{\alpha^2\mu^{\beta+1}t_2^{\beta+1}}{(\beta+1)^2} + \frac{\alpha\mu^{\beta+1}t_2}{(\beta+1)} - \frac{rt_2^3}{6} + \frac{r\mu^2 t_2}{2} - \frac{r\mu^3}{3} - \frac{\alpha\beta r t_2^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{r\alpha\mu^2 t_2^{\beta+1}}{2(\beta+1)} + \frac{\alpha r\beta\mu^{\beta+3}}{(\beta+1)(\beta+3)} - \frac{r\alpha\mu^{\beta+2}t_2}{(\beta+2)} - \frac{r\alpha^2(\beta-2)t_2^{2\beta+3}}{4(\beta+2)(2\beta+3)} + \frac{r\alpha^2\mu^2 t_2^{2\beta+1}}{4(2\beta+1)} + \frac{r\alpha^2(3\beta+1)\mu^{2\beta+3}}{2(\beta+1)(2\beta+3)(2\beta+1)}\right]$$

Total inventory over the period $[0, t_2]$ *is* given by

$$\int_0^{t_2} Q(t)e^{-rt}\, dt =$$

$$(\gamma - 1)\, D_0\left[\frac{t_1^3}{6} - \frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(\beta+2)(2\beta+3)} - \frac{rt_1^4}{8} + \frac{\alpha\beta r t_1^{\beta+4}}{2(\beta+2)(\beta+4)} + \frac{r\alpha^2(3\beta+2)t_1^{2\beta+4}}{2(2\beta+2)(\beta+2)(2\beta+4)}\right] + D_0\left[\frac{\gamma\mu t_1^2}{2} - \frac{\gamma t_1^3}{2} + \frac{\alpha\gamma t_1^{\beta+2}\mu^2}{\beta+2} - \frac{\alpha\gamma\beta t_1^{\beta+3}}{2(\beta+2)(\beta+1)} + \frac{\alpha^2\gamma t_1^{2\beta+2}\mu}{2(2\beta+2)} - \frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\mu^3}{6} + \frac{t_1^3}{6} + \frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)} - \frac{\alpha\gamma\mu^{\beta+1}t_1^2}{2(\beta+1)} - \frac{\alpha^2\gamma\mu^{\beta+1}t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)} - \frac{\alpha^2\gamma(\beta-2)t_1^{2\beta+3}}{4(\beta+1)(\beta+2)} - \frac{r\gamma t_1^2\mu^2}{4} + \frac{r\gamma t_1^4}{4} - \frac{r\alpha\gamma t_1^{\beta+2}\mu^2}{2(\beta+2)} - \frac{r\gamma\alpha^2 t_1^{2\beta+2}\mu^2}{8(\beta+1)} + \right.$$

$$\frac{r\gamma\alpha^2(\beta^2-4\beta-4)t_1^{2\beta+4}}{8(\beta+1)(\beta+2)^2} + \frac{r\mu^4}{8} - \frac{rt_1^4}{8} - \frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)} + \frac{r\alpha\beta t_1^{\beta+4}}{2(\beta+2)(\beta+4)} - \frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{8(\beta+1)(\beta+2)^2} + \frac{r\alpha^2(3\beta+2)\,{t_1}^{2\beta+4}}{8(\beta+2)^2(\beta+1)} + \frac{r\alpha\gamma\mu^{\beta+2}t_1^2}{2(\beta+2)} + \frac{r\alpha^2\gamma t_1^{\beta+2}\mu^{\beta+2}}{(\beta+2)^2}\Bigg] + D_0\mu\left[\frac{t_2^2}{2} - t_2\,\mu + \frac{\mu^2}{2}\right] - \frac{\alpha\mu t_2^{\beta+1}}{(\beta+1)} + \frac{\alpha\beta t_2^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta\mu^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha\mu^{\beta+1}t_2}{(\beta+1)} + \frac{\alpha^2(\beta-1)t_2^{2\beta+2}}{2(2\beta+2)(\beta+1)} - \frac{\alpha^2\mu {t_2}^{2\beta+1}}{2(2\beta+1)} - \frac{\alpha^2(3\beta+1)\mu^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+1)} + \frac{\alpha^2\mu^{\beta+1}{t_2}^{\beta+1}}{(\beta+1)^2} - \frac{rt_2^3}{6} + \frac{r\mu^2 t_2}{2} - \frac{r\mu^3}{3} - \frac{\alpha\beta rt_2^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{r\alpha\mu^2 t_2^{\beta+1}}{2(\beta+1)} + \frac{\alpha r\beta\mu^{\beta+3}}{(\beta+1)(\beta+3)} - \frac{r\alpha\mu^{\beta+2}t_2}{(\beta+2)} - \frac{r\alpha^2(\beta-2)t_2^{2\beta+3}}{4(\beta+2)(2\beta+3)} + \frac{r\alpha^2\mu^2 t_2^{2\beta+1}}{4(2\beta+1)} + \frac{r\alpha^2(3\beta+1)\mu^{2\beta+3}}{2(\beta+1)(2\beta+3)(2\beta+1)}]$$

(39)

The number of deteriorated items over the period [0, t_2] is given by
Production in [0, t_1] - Demand in [0, μ] – Demand in [0, t_2]

$$= \gamma\,\mathrm{D}_0 \int_0^{t_1} te^{-rt}\,dt - \mathrm{D}_0 \int_0^{\mu} te^{-rt}\,dt - \mathrm{D}_0\mu\int_{\mu}^{t_2} e^{-rt}\,dt$$

$$= \gamma\,\mathrm{D}_0\left(\frac{t_1^2}{2} - \frac{rt_1^3}{3}\right) - \mathrm{D}_0\left(\frac{\mu^2}{2} - \frac{r\mu^3}{3}\right) - \mathrm{D}_0\,\mu\left[(t_2-\mu) - r\left(\frac{t_2^2}{2} - \frac{\mu^2}{2}\right)\right] \tag{40}$$

Hence the production cost over the period [0, t_1)]is given by

$$\int_0^{t_1} Kve^{-ru}\,du = \int_0^{t_1}\frac{\alpha_1\gamma}{R^{s-1}}e^{-ru}\,\mathrm{du}$$

$$= \alpha_1\gamma\ D_0^{1-s}\int_0^{t_1} u^{1-s}\,(1-\mathrm{ru})\,\mathrm{du}$$

$$= \alpha_1\gamma\ D_0^{1-s}\left[\frac{t_1^{2-s}}{(2-s)} - \frac{rt_1^{1-s}}{(1-s)}\right] \tag{41}$$

Shortage cost over the period [0, T] is given by

$$\int_{t_2}^{t}\theta(t)dt = -\int_{t_2}^{T} D_0\,\mu[\{t_2-t) - \delta[T-t_2](t_2-t)]dt\,e^{-rt}$$

$$= -D_0\,\mu\left[\left\{Tt_2 - \frac{t_2^2}{2} - \frac{T^2}{2} - \delta\left(\frac{3t_2T^2}{2} - \frac{3t_2^2T}{2} - \frac{T^2}{2} + \frac{t_2^3}{2}\right) - r\frac{T^2t_2}{2} + r\frac{t_2^3}{2} + r\frac{T^3}{3} + \delta r\,(T-t_2)\left(\frac{t_2T^2}{2} - \frac{t_2^3}{6} - \frac{T^3}{3}\right)\right\}\right] \tag{42}$$

Lost sales cost per cycle is

$$\mathrm{LS}= D_0\,\mu\int_{t_2}^{T}(1-e^{-\delta(T-t_2)})\,dt$$

$$\mathrm{LS}= D_0\,\mu\int_{t_2}^{T}\delta(T-t_2)(t_2-t)\,dT$$

Lost sales cost over the period [0,T] is

$$= D_0\,\mu\delta\left[\frac{3T^2t_2}{2} - \frac{3t_2^2T}{2} - \frac{T^3}{2} + \frac{t_2^3}{2} - \frac{5rt_2T^3}{6} - \frac{rt_2^4}{6} + \frac{rTt_2^3}{6} + \frac{rT^4}{3} + \frac{rt_2^2T^2}{2}\right] \tag{43}$$

From (39)(40),(41),(42),(43), the total average inventory cost X of the system is

$$\mathrm{X}= \frac{1}{t_2}[X_1\,\{\,D_0(\gamma-1)[\frac{t_1^3}{6} - \frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(\beta+2)(2\beta+3)} - \frac{rt_1^4}{8} + \frac{\alpha\beta rt_1^{\beta+4}}{2(\beta+2)(\beta+4)} + \frac{r\alpha^2(3\beta+2)t_1^{2\beta+4}}{2(2\beta+2)(\beta+2)(2\beta+4)}] + D_0[\frac{\gamma\mu t_1^2}{2} - \frac{\gamma t_1^3}{2} + \frac{\alpha\gamma t_1^{\beta+2}\mu^2}{\beta+2} - \frac{\alpha\gamma\beta t_1^{\beta+3}}{2(\beta+2)(\beta+1)} + \frac{\alpha^2\gamma t_1^{2\beta+2}\mu}{2(2\beta+2)} - \frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\mu^3}{6} + \frac{t_1^3}{6} + \frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)} - \frac{\alpha\gamma\mu^{\beta+1}t_1^2}{2(\beta+1)} - \frac{\alpha^2\gamma\mu^{\beta+1}t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)} - \frac{\alpha^2\gamma(\beta-2)t_1^{2\beta+3}}{4(\beta+1)(\beta+2)} - \frac{r\gamma t_1^2\mu^2}{4} + \frac{r\gamma t_1^4}{4} - \frac{r\alpha\gamma t_1^{\beta+2}\mu^2}{2(\beta+2)} - \frac{r\gamma\alpha^2 t_1^{2\beta+2}\mu^2}{8(\beta+1)} + \frac{r\gamma\alpha^2(\beta^2-4\beta-4)t_1^{2\beta+4}}{8(\beta+1)(\beta+2)^2} + \frac{r\mu^4}{8} - \frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)} + \frac{r\alpha\beta t_1^{\beta+4}}{2(\beta+2)(\beta+4)} - \frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{8(\beta+1)(\beta+2)^2} + \frac{r\alpha^2(3\beta+2)\,{t_1}^{2\beta+4}}{8(\beta+2)^2(\beta+1)} + \frac{r\alpha\gamma\mu^{\beta+2}t_1^2}{2(\beta+2)} + \frac{r\alpha^2\gamma t_1^{\beta+2}\mu^{\beta+2}}{(\beta+2)^2}] + D_0\mu[\frac{t_2^2}{2} - t_2\,\mu + \frac{\mu^2}{2} - \frac{\alpha\mu t_2^{\beta+1}}{(\beta+1)} + \frac{\alpha\beta t_2^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta\mu^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha\mu^{\beta+1}t_2}{(\beta+1)} + \frac{\alpha^2(\beta-1)t_2^{2\beta+2}}{2(2\beta+2)(\beta+1)} - \frac{\alpha^2\mu{t_2}^{2\beta+1}}{2(2\beta+1)} - \frac{\alpha^2(3\beta+1)\mu^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+1)} + \frac{\alpha^2\mu^{\beta+1}{t_2}^{\beta+1}}{(\beta+1)^2} - \frac{rt_2^3}{6} + \frac{r\mu^2t_2}{2} - \frac{r\mu^3}{3} - \frac{\alpha\beta rt_2^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{r\alpha\mu^2t_2^{\beta+1}}{2(\beta+1)} + \frac{\alpha r\beta\mu^{\beta+3}}{(\beta+1)(\beta+3)} - \frac{r\alpha\mu^{\beta+2}t_2}{(\beta+2)} - \frac{r\alpha^2(\beta-2)t_2^{2\beta+3}}{4(\beta+2)(2\beta+3)} + \frac{r\alpha^2\mu^2t_2^{2\beta+1}}{4(2\beta+1)} + \frac{r\alpha^2(3\beta+1)\mu^{2\beta+3}}{2(\beta+1)(2\beta+3)(2\beta+1)}\} + X_3\{\,\gamma D_0(\frac{t_1^2}{2} - \frac{rt_1^3}{3}) - D_0(\frac{\mu^2}{2} - \frac{r\mu^3}{3}) - D_0\mu((t_2-\mu) - r\left(\frac{t_2^2}{2} - \frac{\mu^2}{2}\right))\} - X_4D_0\,\mu[\left\{Tt_2 - \frac{t_2^2}{2} - \frac{T^2}{2} - \delta\left(\frac{3t_2T^2}{2} - \frac{3t_2^2T}{2} - \frac{T^3}{2} + \frac{t_2^3}{2}\right) - r\frac{T^2t_2}{2} + r\frac{t_2^3}{2} + r\frac{T^3}{3} + \delta r\,(T-t_2)\left(\frac{t_2T^2}{2} - \frac{t_2^3}{6} - \frac{T^3}{3}\right)\right\} + X_5\,D_0\,\mu\delta\{\frac{3T^2t_2}{2} - \frac{3t_2^2T}{2} - \frac{T^3}{2} + \frac{t_2^3}{2} - \frac{5rt_2T^3}{6} - \frac{rt_2^4}{6} + \frac{rTt_2^3}{6} + \frac{rT^4}{3} + \frac{rt_2^2T^2}{2}\} + \alpha_1\gamma\ D_0^{1-s}\{\frac{t_1^{2-s}}{(2-s)} - \frac{rt_1^{1-s}}{(1-s)}\}] \tag{44}$$

$$X_1 = X_{01} + \frac{\mathrm{X}_3'}{\mathrm{n}^{\mathrm{k}}}$$

Where X_1 is continuously deceases over n since $\frac{dX_1}{dn} < 0, n > 0$

$$X_3 = X_{03} + \frac{X_3'}{n^k}$$

Where X_3 is continuously deceases over n since $\frac{dX_3}{dn} < 0, n > 0$

$$X_4 = X_{04} + \frac{X'_4}{n^k}$$

Where X_4 is continuously deceases over n since $\frac{dX_4}{dn} < 0, n > 0$

$$X_5 = X_{05} + \frac{X_5'}{n^k}$$

Where X_5 is continuously deceases over n since $\frac{dX_4}{dn} < 0, n > 0$

Optimum values of t_1 *and* t_2 for minimum average cost are obtained as in Case 1 which gives

$$X_1\left\{(\gamma-1)D_o\left(\frac{t_1^2}{2}-\frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}\right)-\frac{\alpha^2(3\beta+2)t_1^{2\beta+2}}{2(2\beta+2)(\beta+2)}-\frac{rt_1^3}{2}+\frac{\alpha\beta r(3\beta+2)t_1^{\beta+3}}{2(\beta+2)}+\frac{r\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(\beta+2)}\right)+D_o\left(\gamma\mu t_1-\frac{3}{2}\gamma t_1^2+\alpha\gamma\mu^2 t_1^{\beta+1}-\frac{\alpha\gamma\beta(\beta+3)t_1^{\beta+2}}{2(\beta+2)(\beta+1)}+\frac{\alpha^2\gamma t_1^{2\beta+1}\mu}{2}-\frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}+\frac{t_1^2}{2}-\frac{\alpha\gamma\mu^{\beta+1}t_1}{(\beta+1)}-\frac{\alpha^2\gamma\mu^{\beta+1}t_1^{\beta+1}}{(\beta+1)}-\frac{\alpha^2(3\beta+2)t_1^{2\beta+2}}{2(2\beta+2)(\beta+2)}-\frac{\alpha^2\gamma(\beta-2)t_1^{2\beta+2}(2\beta+3)}{4(\beta+1)(\beta+2)}-\frac{r\gamma t_{1\,\mu^2}}{2}+r\gamma t_1^3-\frac{r\alpha\gamma\mu^2 t_1^{\beta+1}}{2}-\frac{r\gamma\alpha^2\mu^2(2\beta+2)t_1^{2\beta+1}}{8(\beta+1)}+\frac{r\gamma\alpha^2(\beta^2-4\beta-4)t_1^{2\beta+3}}{4(\beta+1)(\beta+2)}-\frac{rt_1^2}{2}+\frac{r\alpha\beta t_1^{\beta+3}}{2(\beta+2)}+\frac{r\alpha^2(3\beta+2)t_1^{2\beta+3}}{4(\beta+2)(\beta+1)}+\frac{r\alpha\gamma\mu^{\beta+2}t_1}{(\beta+2)}+\frac{r\alpha^2\gamma t_1^{\beta+1}\mu^{\beta+2}}{(\beta+2)}\right\}+X_3\gamma D_o(t_1-rt_1^2)+\alpha_1\gamma D_o^{1-s}(t_1^{1-s}-rt_1-s)=0 \tag{45}$$

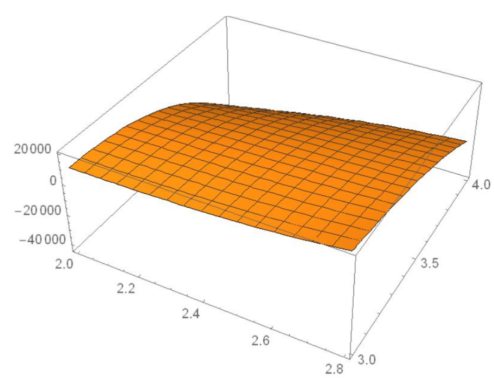
and

$$X_1 D_0\mu\left(t_2-\mu-\alpha\mu t_2^{\beta}+\frac{\alpha\beta t_2^{\beta+1}}{(\beta+1)}+\frac{\alpha\mu^{\beta+1}}{(\beta+1)}+\frac{\alpha^2(\beta-1)t_2^{2\beta+1}}{2(\beta+1)}-\frac{\alpha^2\mu t_2^{2\beta}}{2}+\frac{\alpha^2\mu^{\beta+1}t_2^{\beta}}{(\beta+1)}-\frac{rt_2^2}{2}+\frac{r\mu^2}{2}-\frac{\alpha\beta r t_2^{\beta+2}}{2(\beta+2)}+\frac{r\alpha\mu^2 t_2^{\beta}}{2}-\frac{r\alpha\mu^{\beta+2}}{(\beta+2)}-\frac{r\alpha^2(\beta-2)t_2^{2\beta+2}}{4(\beta+2)}+\frac{r\alpha^2\mu^2 t_2^{2\beta}}{4}\right)-X_3 D_0\mu(1-rt_2)-X_4\, D_0\mu\left\{(T-t_2)-\delta\left(\frac{3T^2}{2}-3t_2T+\frac{3t_2^2}{2}\right)-\frac{rT^2}{2}+\frac{3}{2}rt_2^2+\frac{2}{3}\delta r t_2^3-\delta r t_2 T^2+\frac{5}{6}\delta r T^3-\frac{1}{2}\delta r T t_2^2\right\}+X_5 D_0\mu\delta\left(\frac{3}{2}T^2-3t_2T+\frac{3}{2}t_2^2-\frac{5rT^3}{6}-\frac{2}{3}rt_2^3+\frac{rTt_2^2}{2}+rt_2T^2\right)-X=0 \tag{46}$$

IV. Numerical Examples

Let us consider the inventory system with following data for case I ($\mu \le t_1 \le t_2$)
Data for case I ($\mu \le t_1 \le t_2$)
$D_0 = 14$, $s=1.8$, $\mu=2$, $\alpha_1=2.5$, $\beta= 0.06$, $\alpha=0.08$, $\gamma=2$, $r=0.03$, $X_{01}= 14$, $X_1'=8$, $X_{03}=1.2$, $X_3'=0.2$, $X_{04}=12$, $X_4'=6$, $X_{05}=2.9$, $X_5'=0.6$, $\delta=0.3$, $T=5$, $n=2$, $k=1$
Output results are
$t_1=2.056569$, $t_2= 3.0112976$, T.C. = 203.015

Graphical representation of the converities of t_1 and t_2 w.r.t. T.C for case 1. ($\mu \le t_1 \le t_2$)



Convexity of t_1 *and* t_2 *w.r.t T.C*

TABLE 1: SENSITIVITY ANALYSIS: THE SENSITIVITY ANALYSIS OF THE KEY PARAMETER R, D_0, A, B ARE GIVEN IN THE BELOW TABLE FOR CASE I

PARAMETERS		t_1	t_2	T.C.
r	0.31	2.05202	3.00235	197.976
	0.32	2.04751	2.99348	192.977
	0.33	2.04303	2.98468	188.018
	0.34	2.03859	2.97595	183.098
D_0	14	2.05656	3.01129	203.015
	15	2.05668	3.01125	217.077
	16	2.05677	3.01121	231.16
	17	2.05685	3.01118	245.261
α	0.09	2.08359	2.99942	193.258
	0.10	2.11048	2.98741	183.475
	0.11	2.13719	2.97527	173.67
	0.12	2.16367	2.96302	163.85
β	0.06	2.05656	3.01129	203.015
	0.07	2.05859	3.01060	202.824
	0.08	2.06064	3.00991	202.63
	0.09	2.06270	3.00920	202.434

The Following points are observed

1. $t_1 \delta t_2$ decrease and T.C. also decreases with the increase in value of the parameter r
2. $t_1\delta$T.C. increase while t_2 decreases with the increase in value of the parameter D_0.
3. t_1 increases while t_2 and T.C. decrease with the increase in value of the parameter α.
4. t_1 increases while t_2 and T.C. decrease with the increase in value of the parameter β.

V. CONCLUSION

In this study, an EOQ model with ramp type demand rate and unit production cost under learning and inflationary environment has been developed. The quality and quantity of goods decrease in course of time due to deterioration is a natural phenomena .Hence consideration of Weibull distribution time varying deterioration function defines a significant meaning of perishable, volatile and failure of any kind of item. Shortages are allowed and partially backlogged. The two considered phenomena viz., learning and inflation play an important role in realistic scenario. A mathematical model has been found to determine the optimal ordering policy cost which minimizes the present worth of total optimal cost. Thus the model highlighted the results with numerical examples.

Equation (24) and (25) are non- linear equation in t_1 and t_2. These simultaneous non-linear equations can be solved for suitable choice of the parameters X_1, X_3, X_4, X_5, $n, k, \alpha, \beta, r, \gamma, \mu, \delta$, D_0, α_1 and s ($\neq 2$). If t_1^* and t_2^* are the solution of (24) and (25) for Case I, the corresponding minimum cost $X^*(t_1, t_2)$ can be obtained from (23). It is very difficult to show analytically whether the cost function $X(t_1, t_2)$ is convex. That is why, $X(t_1, t_2)$ may not be global minimum. If $X(t_1, t_2)$ is not convex, then $X(t_1, t_2)$ will be local minimum.

Similarly, solution of equations (45) and (46) for Case II can be obtained corresponding minimum cost $X(t_1, t_2)$ can be obtained from (44).

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