Society of Graphs

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Abstract: At eleven P.M. we say good night. Alas why night and day are complementary but not constant factor for animal kingdom as well as time and space is also a dominating factor. We are trying here to describe an universe of graphs. Where our root is on the earth and head is towards infinity if any. We are introducing the term canopy which encompasses the whole graph theoretic structure under one head. The root of canopy may be zero (of course not well defined) but mind set dictates that the universe may be partially visual.

Keywords: Canopy, Degree Sequence, Reconstruction, label, polynomial, relation, Cut set, circuit, tree, block.

Behind the Bar

Many real world situations can conveniently be described by means of a graph consisting of a set of points together with lines joining certain pairs of these points[1,3,9,10],. A graph is a useful medium for modeling of relevant parts of reality in computer science. A mathematical abstraction of different problem situations of vast area such as operational research, physics, chemistry, economics, genetics, sociology, linguistics, engineering and computer science etc.[6,9,10] give rise to the conceptual versatility of a graph. Any problem which is combinatorial in nature can be solved in terms of graphical structure [3]. In real life, the problems may be of infinite dimensions. Design of ingenious information structure, for minimizing complexity and redundancy of the problem space are versatile. This is unique in the sense given an arbitrary 'n' node graph the problem which becomes countable infinite and in some cases it is uncountable infinite. We have chosen philosophy of Marvin Minsky [9] to take the problem as finite. Which obviously when extended reaches to countable infinite status.
Different areas can be transformation to equivalent graph problems. Problems which can be mapped as graphs are normally simple in nature but we remember the adage "Simple things are mighty things". What we mean that normally for example it is a practice to bring undue mathematics to make things complex but although the graph algorithms are of exponential complexity for large dimension we take it otherwise. Our approach is to make or present simple graph algorithms here, which, when stated in complex way become complex. For example four color conjecture is no longer a conjecture but without any proof, when the graph is planer. The problem is NP-hard [3] even it is 3-coloribility problem just like 3-SAT problem [3]. The polynomial reducibility [3] of 3-SAT to Coloring by forming widget[3,19] was a landmark to see how complex problem can be simplified[3].
Combinatorial algorithms [2] can be defined informally as techniques for the high-speed manipulation of combinatorial objects such as permutation of combinatorial objects such as permutation or graph [2]. The number of such problems are enormous and art of designing such algorithms are especially important and appealing because dealing with combinatorial algorithm is a like playing games or solving puzzles. We are addicted to it. [Don't confuse it with the allurement of lady nicotine.] A good algorithm for combinatorial problems can have a costly payoff has led to terrific advances in the state of the art. According to "Floyd's lemma" [2] the dramatically change of complexity of so called intractable problems [3] now moving to tractable due to the improvement of the algorithms rather than to improvement of hardware.

Representation greatly influence the efficiency of graph algorithms. Linearity can only be achieved through appropriate storage of adjacency information [19]. A good trick design of data structure can enhance the efficiency of designed algorithm for generation of all spanning trees
these are PRIES (Privileged Reduced Incidence Edge Structure) [18] and SPRIES (Super Privileged Reduced Incidence Edge Structure) [17]. According to Cormen and et al. [19] some problems are intractable due to lack of improvement of data structure.

Canopy(n)

When a new term or definition is introduced in graph theory like concept of quantum computing and later a prize winning terminology that may occupy a space in Wikipedia. As observed by Knuth and quoted as " The number of system of terminology presently used in graph theory is equal, to close approximation, to the number of graph theorists.(Richard P. Stanley(1986))[2]". We introduce a "n" integer permutation and combination since that is wonderful, smell like an umbrella in a disturbed atmosphere, a place for small party in a resort or a pandle coverage for a ceremony. The reflection of a tree in the root and the top shows the scenario which is open to expand at any time. The canopy filters the sunlight, softens the rain, and blocks the wind so that younger, more tender life may thrive. In turn, the roots of the trees draw nutrients and moisture from the soil and organic material on the forest floor, conveying it up to the green canopy and keeping it lush and growing.

Similarly, interests and ideas which foster creativity keep people vital and growing. They provide sanctuary from the pressures of life, and allow people to discover who they truly are. Each person brings new perspective to ancient ideas, stretching the boundaries much as the canopy stretches toward the sky.

The canopy is the topmost layer of life which provides shelter for all that resides beneath it.

Definition1:- Canopy (n) is the universal structure consisting of a set of points and their interconnection. It encompasses different sub areas .

For example in terms of 'n' points we can classify different graph theoretic structures following the basic graph theoretic axioms.

Axioms: - i) A set of points n , finite or infinite.

ii) Sum of the interconnections among the points is even.

Canopy has the following structure which may not be finite. We initially give a graphical structure (like family tree) that summarises the present domain of graph theoretic cultural heritage.

We try to define properties of the graph theory as

Properties:-i) adjacency

ii) Incidence

iii) Degree Sequence

Million dollar question- Why another term in graph theory?

We find that scientists search for unified theory in any field for getting a glorified footage. We have met the axioms, theorem, conjecture , lemma in graph algorithms that need to be precisely collected and reformed always as a ready reference. For Example D. E. Knuth when writing his "Art of Computer Programming [2]" used graphs for his future plan or for reading the volumes. His infinite plan shows us a posturize view in volume 4. He should justify to the graph: that he described as the beginning. We think any mathematical term should study as canopy (n) before proceeding further with progenies..

The large number of graph algorithms can be solved in polynomial time if it is labeled graph . In other words we reduce the size of the problem humanologously. Any physical problem can be represented and manipulated in digital computer by codes (discrete, finite).Computer representation of a graph has different fascicles. We are concentrating in a general number sequence usually denary system called degree sequence. Any of the sequence cannot be graphic sequence. They need some additional constraints [5, 16]. After satisfying some constraints it become the degree sequence of a graph i.e. graphic sequence.

In our realization on studying different problem, the degree sequence gives a focus on each of the problem. For example the tree of a graph is the skeleton of the canopy. Hamiltonian path is one of the covers (Like an umbrella).

Graph →Labeled Graph →Reconstruction →Isomorphic→ Isomorphic Graph

According to M.R. Garey / D.S. Johnson [4] most of the intractable problems can be reduced polynomial to graph problems. Degree sequence is a numerical sequence that represents many of the exemplary algorithmic paradigms.

In our deigned figure of Canopy of the graph we tried to cover many of the well-known problems and ambitious that later a large number of problems should be included under this canopy.

Classification

Classification for representation of graph

Canopy is an ancestor with infinitely numerous siblings. Graph is one successor on which our attention is focused now. There are a large variety of parameters to define a graph. Some of them are independent to directly represent the graph and others are combinations of the parameters to represent the graph as unique or any graph. We should not deceive readers the in fact only integers can represent a graph with a variety combinatorial combination of the parameters.

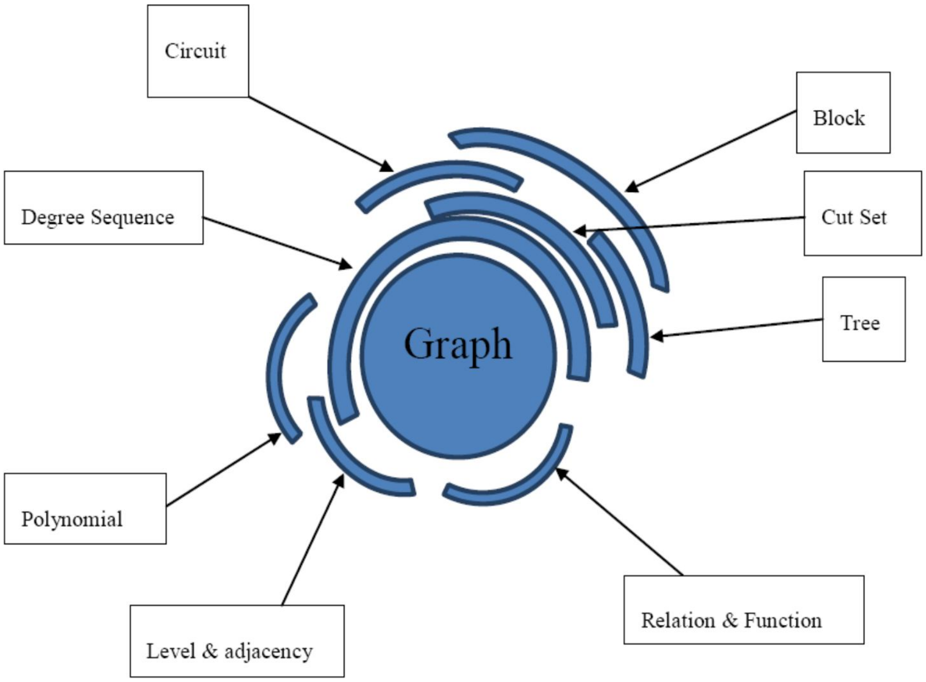

Fig1. Representation of Graph

Relation and Functions [13]
The first one is as relation and functions, where we can define the graph as "A graph G consists of a finite set of vertices and an irreflexive binary relation on vertices. The binary relation may be termed either as a collection of edges of ordered pairs or as a function from vertices to its power set". The classical concept of domain and co-domain is also relevant in our general definition.

Level and adjacency information[13,15
When each vertex are assigned a unique name and the adjacency are mapped as
Adj: V -> p(V) i.e. a function from V to its power set.
We can identify the unique graph using these techniques. A large number of graph theorist problem those are exponential in nature are polynomial solvable for their labeled representation. For example isomorphism checking, Reconstruction etc..

Cut Sets [13]
As we know from set theory that the union of all subset gives the original set. Here we take one special type of sub graph of connected graph whose removal from the original graph separates in two or more sub graphs. Another form of cut set is fundamental cut set [13]. Fundamental Cut Set determines the dimension of the particular graph.

Trees [13]
A tree is a minimally connected sub graph. If we explore the all the trees of a graph, then their commutative sum can give the original graph. The reverse is also true. We have designed various algorithms [18, 20] in different approaches for the reverse circumstances.

Circuit [13, 1]
A walk is defined as a finite alternating sequence of vertices and edges beginning and ending with vertices. Occurrence of vertex may be more than once but edges only once. An open walk is path and a closed walk in which no vertex appears more than once is called a circuit [13].

Block [13]
A connected non-trivial sub graph having no cut point is a block. A block is maximal with respect to his property. If we can get the all blocks of a graph, then we can construct the original graph. Reconstruction conjecture is one of the best examples for these techniques. We have also designed an algorithm for this purpose [15].

Polynomials[1,13,22,15]

Given a non negative integer d, a polynomial in 'm' of degree d is a function P(m) of the form

$P(m)=\sum_{i=0 \text{ to } d} a_i m^i$**;** Where the constraints a_0, a_1,…….,a_d are the coefficient the polynomials and $a_d \neq 0$.

A graph can be expressed elegantly by means of polynomials. There are different types of polynomials those can expressed the different nature of the graph and can be applied to the different graph theoretic problems. Such as Characteristic polynomial [13] for the isomorphism testing of a graph, Chromatic polynomial[13,6] for graph coloring, rank polynomial[13,6] for reconstruction of trees and circuits and matching polynomials[15] are for reconstruction conjecture etc.. These all polynomials are reconstructible. Each representation of the polynomials can represent the unique some specific graphs. Sum of two or more polynomials can represent a unique graph. The polynomials information and degree sequence together can represent the unique graph or up to isomorphism. It can also overlap with the level and adjacency to represent the original graph.

Degree Sequence[1,4,21]

A sequence $d_1,d_2,\ldots\ldots\ldots,d_n$ of non negative integers is called graphical if there's at least one graph on vertices {1,2,…….,n} such that vertex K has degree d_k [2,13].

Degree sequence is an inherent characteristic of any graph. From a non negative integer sequence we can recognize it as graphic sequence or degree sequence after some constraints satisfaction. We can draw a random graph [2] as well as we can identify different characteristics of the graph [7, 8, 17, 19]. If the labeling are added and perform we can draw the graph uniquely. This parameter is much more essential among the others to represent the graph with the help of combinations of other parameters such as cut set, tree, circuit, block and polynomials as shown in the figure1. Degree sequence is acting as pandel coverage of graph or canopy.

Etc.:- For future inclusion of the different representation which can itself or power set of some extra information can represent a graph.

The combinations those are identified in the figure1 can uniquely identify the graph is as

I) Relation and function ii) Level and adjacency iii) Level and degree sequence iv) level, polynomials and degree sequence v) Degree sequence and circuit vi) Degree sequence, cut set and block vii)degree sequence and tree viii) degree sequence, cut set and tree. Ix) Degree sequence, block and tree x) degree sequence, block and circuit xi) degree sequence, cut set and circuit etc.

These combinations are the different parameters for "n" of canopy (n). So we now define the canopy (n) as Canopy (n) as an "n" combinations of parameters identified justly above that encompasses all graphs that may be finite or infinite.

Conclusion

The effort of us to bring all of graphs under same umbrella may be un-imaginable. One may ask why? The answer is simple – one net cannot catch all fishes in a pond. However we advice here with the hope that the last summer is not the lost summer.

References

[1] Arumugam S, Ramachandran S, "Invitation to graph Theory", Scitech Pvt ltd.,2002.

[2] Donald E. Knuth,"The Art of Computer Programming", Volume 1 and 4(Fascicle 0-3), Addison- Wesley, 2009.

[3] Garry M. R. and Johnson D. S., "Computer and Intractability – A guide to the Theory of NP Completeness", W.H. Freeman and Company, 1999.

[4] Gerard Sierksma, Han Hoogeveen, "Seven Criteria for integer sequence being Graphic", Journal of graph theory, Vol. 15, No. 2., pp. 223-231.,1991.

[5] Harary, F. "A Survey of the Reconstruction Conjecture in Graphs and Combinatorics", Lecture Notes in Mathematics, Vol. 406, pp 18-28, 1974.

[6] J. Ebert, "A versatile data structure for edge-oriented graph algorithms", Communication of the ACM, Vol.30, No. 6., pp. 513-519, june.1987.

[7] K.Basuli, S.Naskar, and S.S.Sarma, "Determination of Hamiltonian Circuit and Euler Trail from a Given Degree Sequence", Journal of Physical Science, Volume 12, 2008, pp.- 249-252.

[8] K. Basuli, S. Naskar, and S.S.Sarma, "Role of Degree Sequence in Determination of Maximal Clique of a Graph", Journal of Global Research in Computer Science, Vol.1, No.2, ISSN-2229- 371X, October 2010.

[9] Marvin Lee Minsky.," Computation: Finite and Infinite Machines", PHI. , Jun-1967.

[10] M.C.Golumbic, "Algorithmic Graph Theory and Perfect Graphs", Academic Press, New York, 1980

[11] N. Deo, Graph Theory with Applications to Engineering and Computer Science. The Prentice Hall of India Private Limited, New Delhi, 2003.

[12] Reingold, Nievergelt and Deo, Combinatorial Algorithms: Theory and Practice, PHI, 1977.

[13] S.Banerjee,D.Sensarma,K.Basuli,S.Naskar,S.SenSarma,"From Reconstruction Conjecture Towards A Lemma", Computer Science & Engg.: An International Journal(CSEIJ), Vol.2,No.1,February 2012,pp.-53-65.

[14] S.L. Hakimi., "On realizability of a set of integers as degree of the vertices of a linear graph", Journal of the Society for Industrial and applied mathematics, SIAM, Vol.10. No. 3.,pp. 496-506,Sept.1962.

[15] S. Naskar, K. Basuli, and S.S.Sarma,"Connectedness of a Graph from its Degree Sequence and it is Relevent with Reconstruction Conjecture",Journal of Global Research in Computer Science, Vol.1, No.1, ISSN-2229-371X, August 2010.

[16] S.Naskar, K.Basuli, and S.S.Sarma "When a Degree Sequence represents a Tree Sequence", ICFAI University Journal of Computer Science, Volume 2, No. 3, p.p.-62-69, 2008.

[17] S. Naskar, K. Basuli, and S.S.Sarma,"Generation of All Spanning Trees in the Limelight"proceedings of The Second International Conference, CCSIT 2012,Part-III, Bangalore, Springer , India,pp.188-192

[18] Sarma Sen S., et al, An efficient tree generation algorithm, Journal of Institution of Electronics and Telecommunications Engg.(IETE),Vol 27,No 3,pp 105-109,1981.

[19] Thomas. H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford stein, " Introduction to algorithm", PHI, New Delhi, 2008.

[20] Ulam, S "A Collection of Mathematical Problems". Inter science Publishers, New York, 1960. USA.